Answer Key

Pi in the Sky 9

The Lunar Flashlight spacecraft shines a laser into a dark crater on the Moon, lighting up a circular area on the surface. Earth sits just beyond the horizon on the Moon's south pole, a perspective that makes Earth appear as if it has been flipped 180 degrees. Image credit: NASA/JPL-Caltech | + Expand image

Lunar Logic

NASA's Lunar Flashlight mission will observe and map the location of frost within permanently shadowed craters in the Moon's south polar region. Knowing how much frost is in these craters and where to find it can help us prepare for extended missions on the Moon, when water will be a valuable resource.

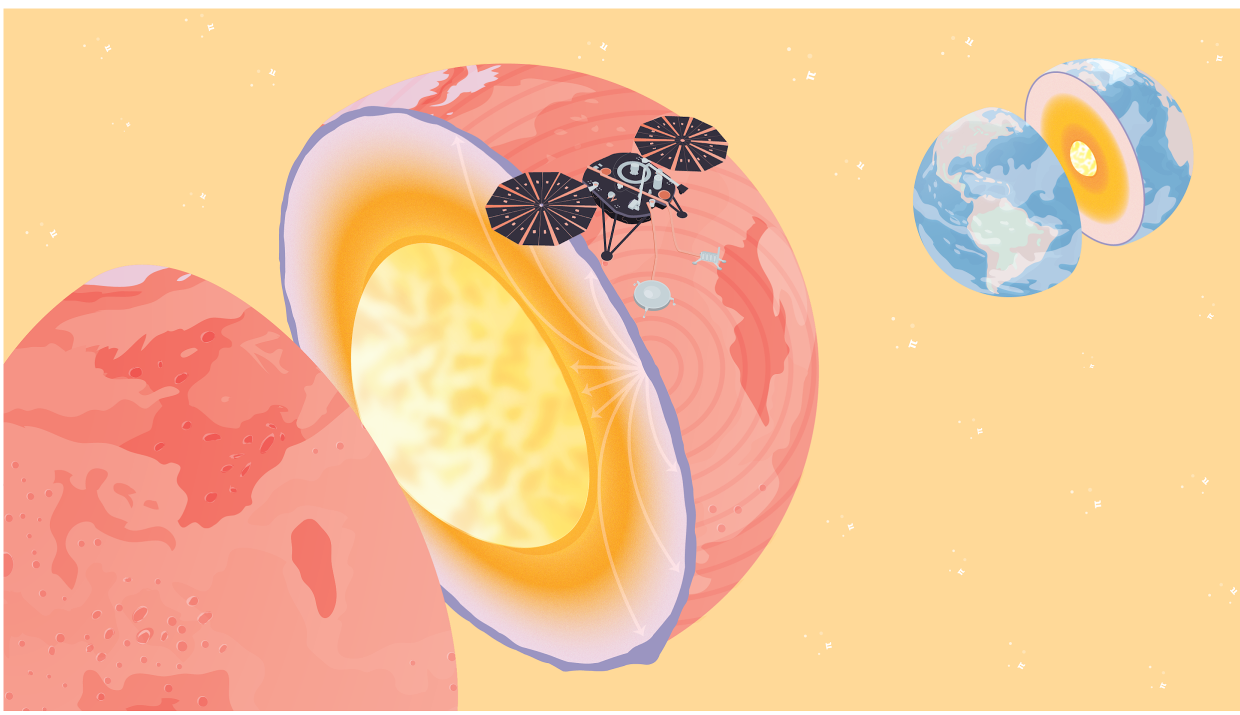
The spacecraft, a backpack-size cubesat, will collect data during 10 orbits over a two-month period, making repeated measurements over multiple points to map ice in these dark craters. To take measurements, Lunar Flashlight will send infrared laser pulses to the surface of the Moon and measure the signal that is reflected. The amount of light that is reflected back will help scientists determine where the lunar surface is dry and where it contains water-ice.

At 20 km altitude, the spacecraft's infrared lasers have a radius of 17.5 meters when they reach the surface of the Moon.

How much area do they cover in a single pulse?

1. Use the formula for area of a circle to calculate the area covered by a laser pulse.
   $A = \pi r^2$
   $A = \pi(17.5m)^2$
   $A = \pi(306.25\ m^2)$
   $A \approx$ **$962\ m^2$**

› Learn more about the Lunar Flashlight mission



The InSight lander is shown on the surface of Mars, where circular lines radiate out from a central point. The interior of Mars is shown with lines flowing left and right from the same central point and extending from the crust into Mars' mantle down to its large central core. In the background, a cutaway shows the interior of Earth with more interior layers and a smaller core. Image credit: NASA/JPL-Caltech | + Expand image

Core Conundrum

The InSight Mars lander is equipped with several tools to help scientists learn more about the interior of the Red Planet, including a seismometer that detects marsquakes. By measuring the vibrations that travel across the surface of Mars and through its interior layers, scientists were able to accurately measure the size of Mars' liquid core and estimate its density. Knowing the size and density of Mars' core will help us learn more about how the planet formed, how its magnetic field developed, and what materials make up the core, which will ultimately lead to a better understanding of how Earth and other planets form.

If Mars' core has a mass of 1.54 x 10^{23} kg and a radius of 1,830 km, as measured by InSight, what is the density of the core?

1. Convert km to cm.
   1,830 km * (100,000 cm/1 km) = 183,000,000 cm = $1.83*10^8$ cm

2. Calculate the volume of Mars' core.
   $V = 4/3\pi r^3$
   $V = 4/3\pi(1.83 * 10^8 \text{ cm})^3$
   $V \approx 4/3\pi(6.13 * 10^{24} \text{ cm})$
   $V \approx 2.57 * 10^{25} \text{ cm}^3$

3. Convert kg to g.
   $(1.54*10^{23}$ kg) * (1,000 g / 1kg) = $1.54*10^{26}$ g

4. Divide the mass of Mars' core by its volume.
   $(1.54*10^{26}$ g) / $(2.57*10^{25}$ cm^3) ≈ **5.99 g/cm^3**

How does that compare to the density of Earth's core, which ranges from 10 to 13 g/cm^3?

Mars' core is less dense.

What does that tell us about the makeup of Mars' core?

Mars' core is made of less dense material than Earth's core.

› Learn more about the InSight lander

In this whimsical tropical scene, triangular radar beams extend down and bounce back up from either side of the SWOT spacecraft as it flies over a reservoir blocked by a dam. Water flows from one of the cylindrical pipes set into the wall of the dam. Image credit: NASA/JPL-Caltech | + Expand image

Dam Deduction

Water exiting a hydropower dam is called non-powered or powered outflow. Non-powered outflow exits via a spillway, on top of the dam. Powered outflow, which is used to generate electricity, travels through penstocks, pipes at the bottom of a dam. Powered outflow is usually colder and travels at a higher velocity, so it can disturb sediments, temperatures, and water quality of downstream rivers, especially when it's a high percentage of the total outflow.

The SWOT mission, a satellite designed to survey all of Earth's surface water, including lakes, rivers, oceans, and reservoirs, can help scientists better analyze these impacts.

A dam has 3 penstocks with diameters of 6.2 meters and a measured total outflow of 1,350 m^3/s. If SWOT measured the reservoir's water depth (H) at 100 m above the penstocks, **compute the velocity (m/s) of the powered outflow using V=√2gH.**

1. Plug in the values for the measured height of the reservoir (H) above the penstocks and acceleration of gravity constant (g), and compute.

   $V = \sqrt{2gH} = \sqrt{2(9.8\, m/s^2)(100\, m)} = \sqrt{1,960\, m^2/s^2}$ **≈ 44 m/s**

What is the powered outflow if 1 penstock is open?

1. Compute the area of a penstock opening and multiply by the velocity.

   $A = \pi r\^2 = 3.14 * (3.1\, m)^2 \approx 30\ m^2$
   Powered outflow ≈ ($30\ m^2$)(44 m/s) ≈ **$1,320\ m^3/s$**

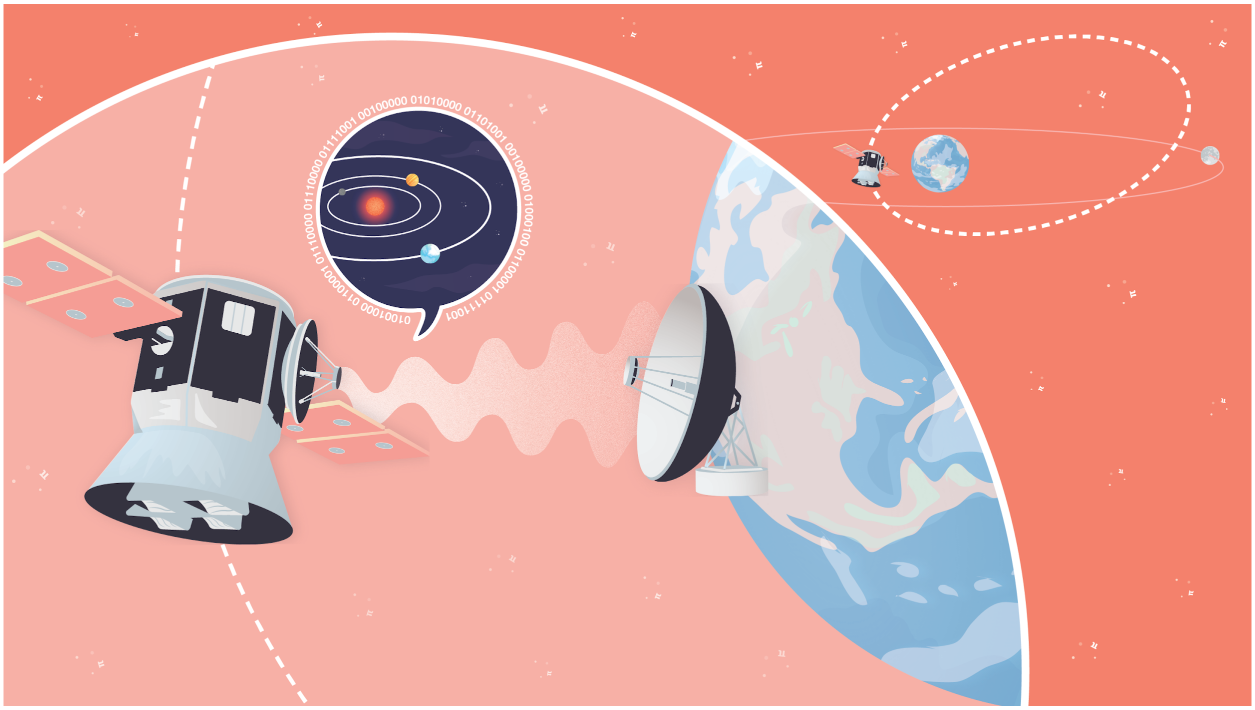
Is this a high or low percentage of the total outflow?

1. Compute the ratio of powered outflow to total outflow.
   ($1,320\ m^3/s$)/($1,350\ m^3/s$) **≈ 98%, a high percentage of total outflow**

What can this tell you about the potential environmental impacts?

The potential for environmental impact is high.

› Learn more about the SWOT mission



The TESS spacecraft's elliptical orbit is shown in a faraway view with the Moon orbiting Earth. TESS' orbit brings it close to Earth, then down below the plane of the Moon's orbit, and then way up above. In a closer view, a snake-like shape flows from TESS' antenna to a larger antenna on Earth to represent the spacecraft sharing the data it has collected. A smaller inset represents the data itself – the discovery of an exoplanet system around a red dwarf star. Image credit: NASA/JPL-Caltech | + Expand image

Telescope Tango

NASA's TESS mission is designed to survey the entire sky in search of exoplanets, or planets orbiting stars other than our Sun. In its two-year primary mission, TESS identified more than 2,600 possible exoplanets and counting.

To locate exoplanets, the space telescope flies in a highly eccentric elliptical orbit, which had never been attempted before. This orbit, called P/2, minimizes the amount of time that light and heat from Earth and the Moon can interfere with data collection. And it still allows the spacecraft to make close passes by Earth to transmit data about its findings back to scientists. The spacecraft's 13.7 day orbit has an axis of 376,000 km at apogee and an axis of 108,400 km at perigee. Each downlink from TESS takes about three hours to complete.

While TESS actually moves at different speeds throughout its orbit – from 0.5 km/s at apogee to 4 km/s at perigee – **if its velocity stayed uniform, how many kilometers would TESS need to travel to successfully transmit its data?**

1. Plug in the values for the semi-major axis (apogee axis/2) and the semi-minor axis (perigee axis/2) into the equation for the perimeter of an ellipse to find the total distance TESS travels throughout its orbit.
   P ≈ $\pi[3(a + b) - \sqrt{(3a + b)(a + 3b)}]$
   P ≈ $\pi[3(188,000km + 54,200km) -$
   $\sqrt{((3 * 188,000km) + 54,200\ km)(188,000km + (3 * 54,200km))}]$
   P ≈ **820,100 km**

2. Divide the downlink time by the time it takes TESS to complete its orbit to find the percentage of the orbit spent sending data back to Earth.
   3 hours / (13.7 days * 24 hours) ≈ **0.9% of perimeter**

3. Multiply the percent of time transmitting by the total perimeter to get the distance covered in this time:
   0.009* 820,100 km ≈ **7,380 km**

   ***Note:** There are many ways to solve this problem. One way is to use the Ramanujan approximation as shown above. However, calculus can also be used.

› Learn more about the TESS mission