# UNIT 8 Geometry

Many artists use geometric concepts in their work.

Think about what you have learned in geometry. How do these examples of First Nations art and architecture show geometry ideas?

## What You'll Learn

- Identify and construct parallel and perpendicular line segments.
- Construct perpendicular bisectors and angle bisectors, and verify the constructions.
- Identify and plot points in the four quadrants of a grid.
- Graph and describe transformations of a shape on a grid.

## Why It's Important

- A knowledge of the geometry of lines and angles is required in art and sports, and in careers such as carpentry, plumbing, welding, engineering, interior design, and architecture.

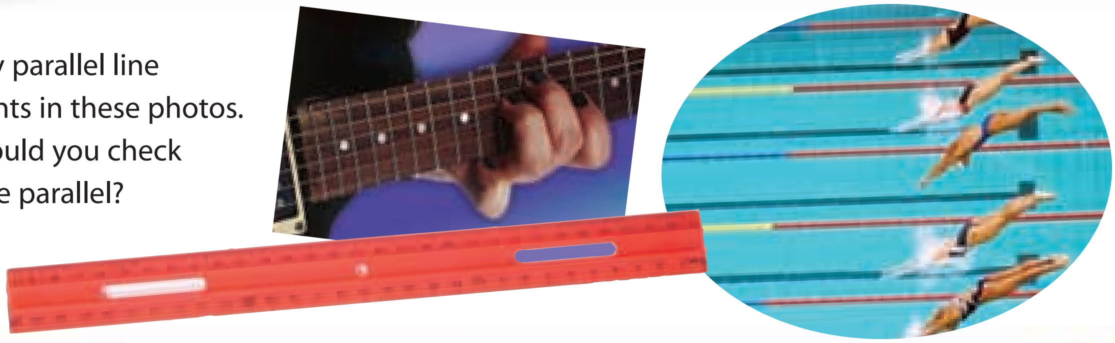
## Key Words

- parallel lines
- perpendicular lines
- line segment
- bisect
- bisector
- perpendicular bisector
- angle bisector
- coordinate grid
- Cartesian plane
- *x*-axis
- *y*-axis
- origin
- quadrants

# 8.1 Parallel Lines

**Focus** Use different methods to construct and identify parallel line segments.

Identify parallel line segments in these photos. How could you check they are parallel?



## Explore

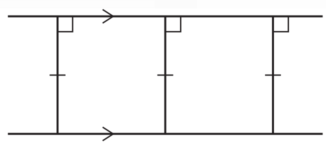
You may need a ruler, plastic triangle, tracing paper, protractor, and Mira.
Use any methods or tools. Draw a line segment on plain paper.
Draw a line segment parallel to the line segment.
Find as many ways to do this as you can using different tools.

### Reflect & Share

Compare your methods with those of another pair of classmates.
How do you know the line segment you drew is parallel to the line segment?
Which method is most accurate? Explain your choice.

## Connect

**Parallel lines** are lines on the same flat surface that never meet.
They are always the same distance apart.



Here are 3 strategies to draw a line segment parallel to a given line segment.

➤ Use a ruler.
Place one edge of a ruler along the line segment.
Draw a line segment along the other edge of the ruler.

➤ Use a ruler and protractor.
Choose a point on the line segment.
Place the centre of the protractor on the point.

Align the base line of the protractor with the line segment.
Mark a point at 90°.
Repeat this step once more.
Join the 2 points to draw a line segment parallel to the line segment.

➤ Use a ruler and compass as shown below.

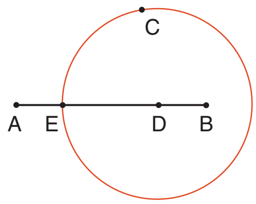
## Example

Use a ruler and compass to draw a line segment parallel to line segment AB that passes through point C.

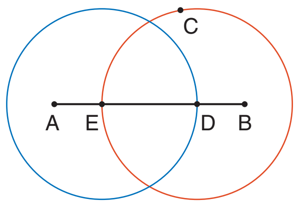
A *line segment* is the part of a line between two points on the line.

### A Solution

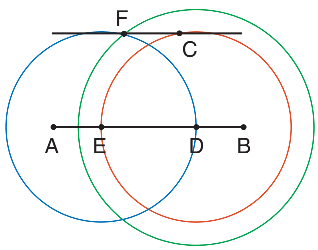
- Mark any point D on AB.




- Place the compass point on D.
  Set the compass so the pencil point is on C.
  Draw a circle.
  Label point E where the circle intersects AB.
- Do not change the distance between the compass and pencil points.
  Place the compass point on E.
  Draw a circle through D.




- Place the compass point on E.
  Set the compass so the pencil point is on C.
- Place the compass point on D.
  Draw a circle to intersect the circle through D.
  Label the point of intersection F.

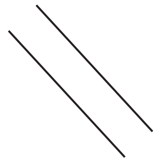


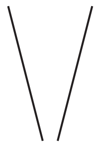

- Draw a line through points C and F.
  Line segment CF is parallel to AB.

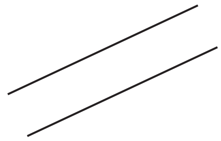
The 2 line segments are parallel because they are always the same distance apart.

# Practice

**1.** Which lines are parallel? How do you know?

a) 

b) 

c) 

**2.** a) Draw line segment CD of length 5 cm.
Use a ruler to draw a line segment parallel to CD.

b) Choose 3 different points on CD.
Measure the shortest distance from each point to the line segment you drew.
What do you notice?

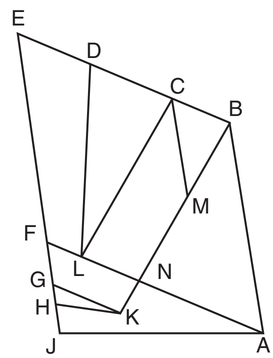
**3.** Draw line segment EF of length 8 cm.

a) Use a ruler and protractor to draw a line segment parallel to EF.

b) Use a ruler and compass to draw a line segment parallel to EF.

**4.** Suppose there are 2 line segments that look parallel.
How could you tell if they are parallel?

**5.** Make a list of where you see parallel line segments in your community or around the house.
Sketch diagrams to illustrate your list.




**6.** **Assessment Focus** Your teacher will give you a large copy of this diagram.
Find as many pairs of parallel line segments as you can.
How do you know they are parallel?

**7.** **Take It Further** Draw line segment CD.
Use what you know about drawing parallel line segments to construct parallelogram CDEF.
Explain how you can check you have drawn a parallelogram.

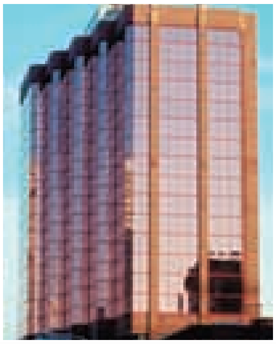
## Reflect

Describe 3 different methods you can use to draw a line segment parallel to a given line segment. Which method do you prefer?
Which method is most accurate? Explain your choice.

# 8.2 Perpendicular Lines

**Focus** Use different methods to construct and identify perpendicular line segments.

Identify perpendicular line segments in these photos. How could you check they are perpendicular?



## Explore

You may need a ruler, plastic triangle, protractor, and Mira.
Use any methods or tools. Draw a line segment on plain paper.
Draw a line segment perpendicular to the line segment.
Find as many ways to do this as you can using different tools.

### Reflect & Share

Compare your methods with those of another pair of classmates. How do you know the line segment you drew is perpendicular to the line segment?
Which method is most accurate? Explain your choice.

**Recall that 2 lines intersect if they meet or cross.**

## Connect

Two line segments are **perpendicular** if they intersect at right angles.
Here are 5 strategies to draw a line segment perpendicular to a given line segment.

- Use a plastic right triangle.
  Place the base of the triangle along the line segment.
  Draw a line segment along the side that is the height of the triangle.

- Use paper folding.
  Fold the paper so that the line segment coincides with itself. Open the paper.
  The fold line is perpendicular to the line segment.

➤ Use a ruler and protractor.
Choose a point on the line segment.
Place the centre of the protractor on the point.
Align the base line of the protractor with
the line segment. Mark a point at 90°.
Join the 2 points to draw a line segment
perpendicular to the line segment.

➤ Use a Mira. Place the Mira so that the reflection
of the line segment coincides with itself
when you look in the Mira.
Draw a line segment along the edge of the Mira.

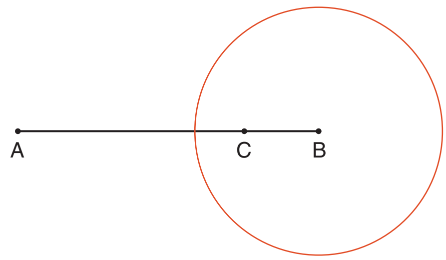
➤ Use a ruler and compass as shown below.

## Example

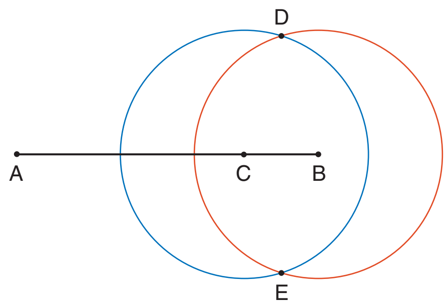
Use a ruler and compass to draw a line segment
perpendicular to line segment AB.

### A Solution

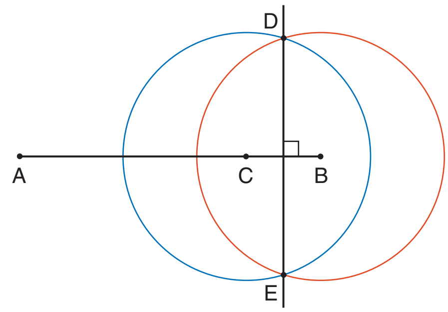
- Mark a point C on AB.




- Set the compass so the distance between the compass and pencil points is greater than one-half the length of CB. Place the compass point on B. Draw a circle that intersects AB.
- Do not change the distance between the compass and pencil points. Place the compass point on C. Draw a circle to intersect the first circle you drew. Label the points D and E where the circles intersect.



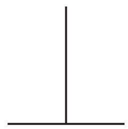

- Draw a line through points D and E. DE is perpendicular to AB.

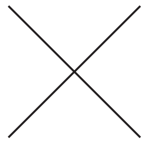


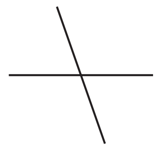

To check, measure the angles
to make sure each is 90°.

## Practice

1. Which lines are perpendicular? How do you know?

a) 

b) 

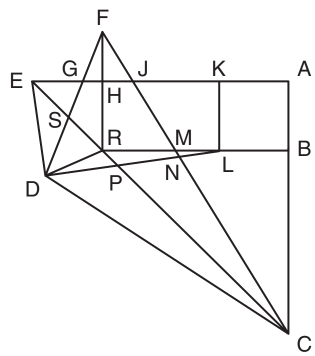
c) 

2. a) Draw line segment AB of length 6 cm.
Use a Mira to draw a line segment perpendicular to AB.
b) Draw line segment CD of length 8 cm. Mark a point on the segment.
Use paper folding to construct a line segment perpendicular to CD that passes through the point.
How do you know that each line segment you drew is perpendicular to the line segment?

3. Draw line segment EF of length 10 cm.
a) Use a ruler and protractor to draw a line segment perpendicular to EF.
b) Use a ruler and compass to draw a line segment perpendicular to EF.
c) Check that the line segments you drew are perpendicular to EF.

4. Make a list of where you see perpendicular line segments in the world around you. Sketch diagrams to illustrate your list.

5. **Assessment Focus** Your teacher will give you a large copy of this diagram.
Find as many pairs of perpendicular line segments as you can.
How do you know they are perpendicular?




6. **Take It Further** Draw line segment JK of length 10 cm.
Use what you know about drawing perpendicular and parallel line segments to construct a rectangle JKMN, where KM is 4 cm.
Explain how you can check you have drawn a rectangle.

## Reflect

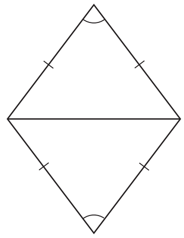
Describe 4 different methods you can use to draw a line perpendicular to a given line segment.
Which method do you prefer?
Which method is most accurate? Explain your choice.

# 8.3 Constructing Perpendicular Bisectors

**Focus** Use a variety of methods to construct perpendicular bisectors of line segments.

Recall that a rhombus has all sides equal and opposite angles equal.

Each diagonal divides the rhombus into 2 congruent isosceles triangles.
How do you know the triangles are isosceles?
How do you know the triangles are congruent?



You will investigate ways to cut line segments into 2 equal parts.

## Explore

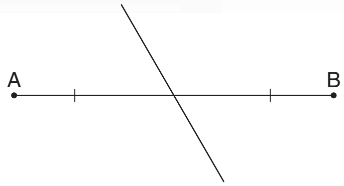
You may need rulers, protractors, tracing paper, plain paper, and Miras.
Use any methods or tools.
Draw a line segment on plain paper.
Draw a line segment perpendicular to the line segment that cuts the line segment in half.

### Reflect & Share

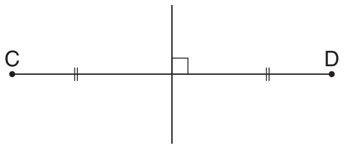
Compare your results and methods with those of another pair of classmates.
How could you use your method to cut your classmate's line segment in half?

## Connect

When you draw a line to divide a line segment into two equal parts, you **bisect** the segment.
The line you drew is a **bisector** of the segment.




When the bisector is drawn at right angles to the segment, the line is the **perpendicular bisector** of the segment.

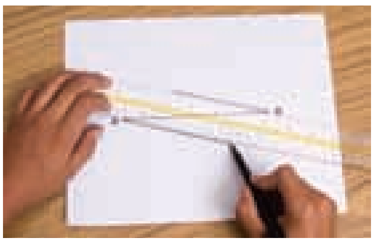


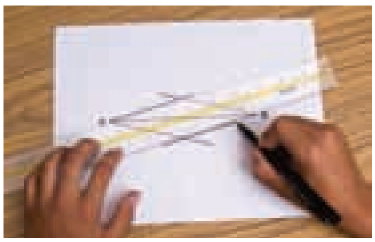
Here are 3 strategies to draw the perpendicular bisector of a given line segment.

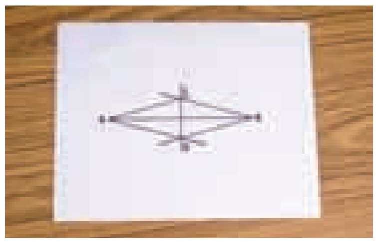

➤ Use paper folding.
Fold the paper so that point A lies on point B.
Crease along the fold. Open the paper.
The fold line is the perpendicular bisector of AB.

➤ Use a Mira. Place the Mira so that the reflection of point A lies on point B.
Draw a line segment along the edge of the Mira.

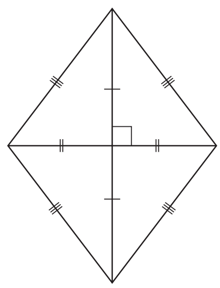
➤ Use a ruler. Place the ruler so that A is on one side of the ruler and B is on the other.
Draw line segments along both edges of the ruler.
Repeat this step once more so that A and B are now on opposite sides of the ruler.
Draw line segments along both edges of the ruler.
Label the points C and D where the line segments you drew intersect.
Join CD. CD is the perpendicular bisector of AB.







Recall that each diagonal of a rhombus is a line of symmetry.
The diagonals intersect at right angles.
The diagonals bisect each other.
So, each diagonal is the perpendicular bisector of the other diagonal.



We can use these properties of a rhombus to construct the perpendicular bisector of a line segment.
Think of the line segment as a diagonal of a rhombus.
As we construct the rhombus, we also construct the perpendicular bisector of the segment.

## Example

Use a ruler and compass to draw the perpendicular bisector of any line segment AB.

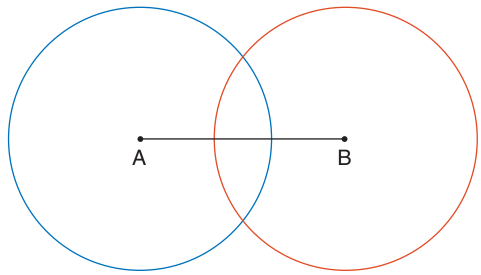
## A Solution

Use a ruler and compass.

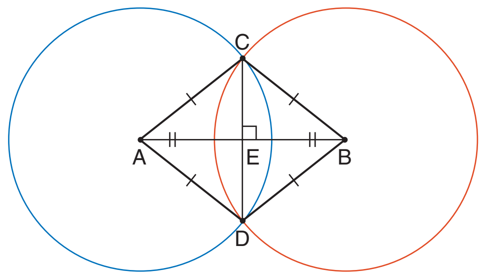
- Draw any line segment AB.


- Set the compass so the distance between the compass and pencil points is greater than one-half the length of AB.

- Place the compass point on A.
  Draw a circle.
  Do not change the distance between the compass and pencil points.
  Place the compass point on B.
  Draw a circle to intersect the first circle you drew.




- Label the points C and D where the circles intersect.
  Join the points to form rhombus ACBD.
  Draw the diagonal CD.
  The diagonals intersect at E.
  CD is the perpendicular bisector of AB.
  That is, AE $=$ EB and $\angle$AEC $=$ $\angle$CEB $=$ 90°




To check that the perpendicular bisector has been drawn correctly, measure the two parts of the segment to check they are equal, and measure the angles to check each is 90°.

**Note that any point on the perpendicular bisector of a line segment is the same distance from the endpoints of the segment.**
**For example, AC $=$ BC and AD $=$ BD**

## Practice

Show all construction lines.

**1.** a) Draw line segment CD of length 8 cm.
Use paper folding to draw its perpendicular bisector.

b) Choose three different points on the bisector.
Measure the distance to each point from C and from D.
What do you notice?

**2.** a) Draw line segment EF of length 6 cm.
Use a Mira to draw its perpendicular bisector.
b) How do you know that you have drawn the perpendicular bisector of EF?

**3.** Draw line segment GH of length 4 cm.
Use a ruler to draw its perpendicular bisector.

**4.** a) Draw line segment AB of length 5 cm.
Use a ruler and compass to draw its perpendicular bisector.
b) Choose three different points on the bisector.
Measure the distance to each point from A and from B.
What do you notice? Explain.

**5.** Find out what happens if you try to draw the perpendicular bisector of a line segment when the distance between the compass and pencil points is:
a) equal to one-half the length of the segment
b) less than one-half the length of the segment

**6.** **Assessment Focus** Draw line segment RS of length 7 cm.
Use what you know about perpendicular bisectors to construct rhombus RTSU.
How can you check that you have drawn a rhombus?

**7.** Look around you. Give examples of perpendicular bisectors.

**8.** **Take It Further** Draw a large △PQR.
Construct the perpendicular bisector of each side.
Label point C where the bisectors meet.
Draw the circle with centre C and radius CP.

**"Circum" is Latin for "around." So, the *circumcircle* is the circle that goes around a triangle.**

**9.** **Take It Further**
a) How could you use the construction in question 8 to draw a circle through any 3 points that do not lie on a line?
b) Mark 3 points as far apart as possible. Draw a circle through the points. Describe your construction.

**The point at which the perpendicular bisectors of the sides of a triangle intersect is called the *circumcentre*.**

## Reflect

How many bisectors can a line segment have?
How many perpendicular bisectors can a line segment have?
Draw a diagram to illustrate each answer.

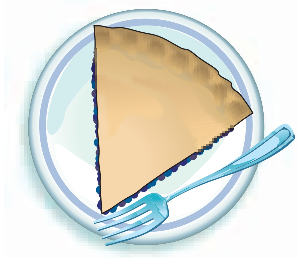
# 8.4 Constructing Angle Bisectors

**Focus** Use a variety of methods to construct bisectors of angles.

You will investigate ways to divide an angle into 2 equal parts.

## Explore

Your teacher will give you a large copy of this picture.
You may need rulers, protractors, tracing paper, plain paper, and Miras.
Use any methods or tools.
George wants to share this slice of pie equally with a friend.
Show how he could divide the slice of pie into 2 equal parts.



### Reflect & Share

Compare your results and methods with those of another pair of classmates.
How could you use your classmates' methods to divide the slice of pie in half?

## Connect

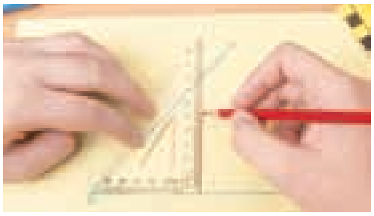
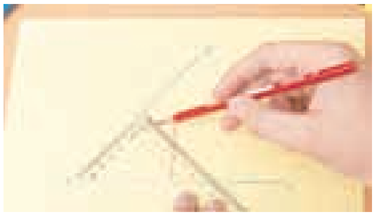
When you divide an angle into two equal parts, you *bisect* the angle.

Here are 3 strategies to draw the bisector of a given angle.

➤ Use paper folding.
Fold the paper so that XY lies along ZY.
Crease along the fold line.
Open the paper.
The fold line is the bisector of ∠XYZ.

➤ Use a Mira. Place the Mira so that the reflection of one arm of the angle lies along the other arm.
Draw a line segment along the edge of the Mira.
This line segment is the bisector of the angle.

➤ Use a plastic right triangle.
Place the triangle with one angle at B and one edge along BC.
Draw a line segment.
Place the triangle with the same angle at B and the same edge along AB.
Draw a line segment.
Label M where the line segments you drew intersect. BM is the bisector of ∠ABC.

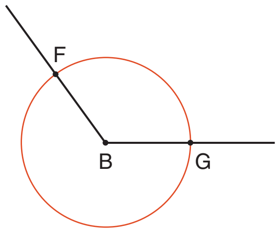

We can use the properties of a rhombus to construct the bisector of an angle.
Think of the angle as one angle of a rhombus.

## Example

Draw obtuse ∠B of measure 126°.
Use a ruler and a compass to bisect the angle.
Measure the angles to check.

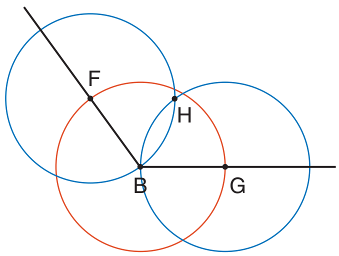
## A Solution

Use a ruler and protractor to draw ∠B = 126°.
Use ∠B as one angle of a rhombus.
With compass point on B, draw a circle that intersects one arm at F and the other arm at G.

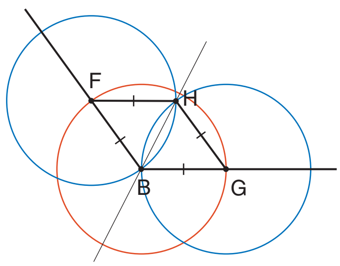



**FB and BG are 2 sides of the rhombus; FB = BG**

Do not change the distance between the compass and pencil points.
Place the compass point on F.
Draw a circle.
Place the compass point on G.
Draw a circle to intersect the second circle you drew.
Label the point H where the circles intersect.




Join FH and HG to form rhombus BFHG.
Draw a line through BH.
This line is the **angle bisector** of ∠FBG.
That is, ∠FBH = ∠HBG




**FH and HG are the other 2 sides of the rhombus.**

**BH is a diagonal of the rhombus.**

Use a protractor to check. Measure each angle.

∠FBG = 126°

∠FBH = 63° and ∠GBH = 63°

∠FBH + ∠GBH = 63° + 63°

= 126°

= ∠FBG

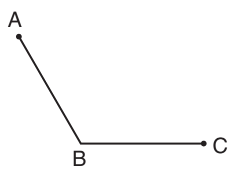
To check that the bisector of an angle has been drawn correctly, we can:

- Measure the two angles formed by the bisector. They should be equal.
- Fold the angle so the bisector is the fold line. The two arms should coincide.
- Place a Mira along the angle bisector. The reflection image of one arm of the angle should coincide with the other arm, and vice versa.

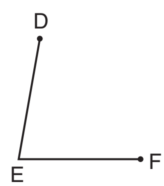
## Practice

Show any construction lines.

**1.** Your teacher will give you a copy of this obtuse angle.
Use a Mira to bisect the angle.
Measure the two parts of the angle.
Are they equal?




**2.** Your teacher will give you a copy of this acute angle.
Use a plastic right triangle to bisect the angle.
Measure the two parts of the angle.
Are they equal?




**3.** Use a ruler and compass.

a) Draw acute ∠PQR = 50°. Bisect the angle.

b) Draw obtuse ∠GEF = 130°. Bisect the angle.

**4.** Draw a reflex angle of measure 270°.

a) How many different methods can you find to bisect this angle?

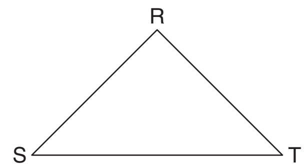
b) Describe each method.
Check that the bisector you draw using each method is correct.

A ***reflex angle*** is an angle between 180° and 360°.

**5.** You have used Miras, triangles, and paper folding to bisect an angle. What is the advantage of using a ruler and compass?

**6.** a) Draw line segment HJ of length 8 cm.
Draw the perpendicular bisector of HJ.
b) Bisect each right angle in part a.
c) How many angle bisectors did you need to draw in part b?
Explain why you needed this many bisectors.

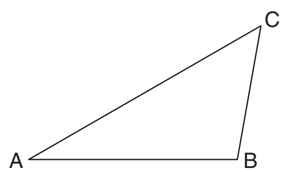
**7.** **Assessment Focus** Your teacher will give you a large copy of this isosceles triangle. Use a ruler and compass.




a) Bisect ∠R.
b) Show that the bisector in part a is the perpendicular bisector of ST.
c) Is the result in part b true for:
i) a different isosceles triangle?
ii) an equilateral triangle?
iii) a scalene triangle?
How could you find out? Show your work.

**8.** Describe examples of angle bisectors that you see in the environment.

**9.** **Take It Further** Your teacher will give you a copy of this triangle.
Cut it out.
Fold the triangle so BC and BA coincide. Open the triangle.
Fold it so AB and AC coincide. Open the triangle.
Fold it so AC and BC coincide. Open the triangle.




a) Measure the angles each crease makes at each vertex.
What do you notice?
b) Label point K where the creases meet.
Draw a circle in the triangle that touches each side of △ABC.
What do you notice?
c) What have you constructed by folding?

## Reflect

How many bisectors can an angle have?
Draw a diagram to illustrate your answer.

# Mid-Unit Review

LESSON

8.1

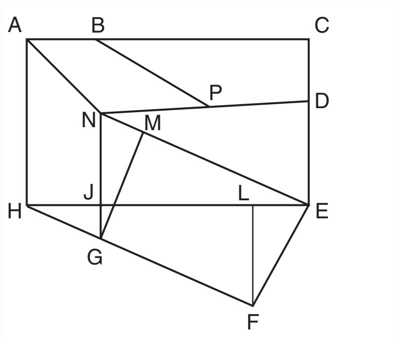
**1.** a) Draw a line segment CD of length 9 cm. Draw a point F not on the line segment. Use a ruler and compass to construct a line segment parallel to CD that passes through F.

b) Draw line segment CD again. Use a different method to construct a line segment parallel to CD that passes through F.

c) Which method is more accurate? Explain you choice.

8.2

**2.** Your teacher will give you a large copy of this picture.




a) Identify as many parallel line segments as you can. How do you know they are parallel?

b) Find as many perpendicular line segments as you can. How do you know they are perpendicular?

8.3

**3.** a) Draw line segment AB of length 10 cm. Use a ruler and compass to draw the perpendicular bisector of AB.

b) Draw line segment AB again. Use a different method to draw the perpendicular bisector.

c) How can you check that you have drawn each bisector correctly?

**4.** a) Draw line segment AB of length 6 cm. AB is the base of a triangle.

b) Construct the perpendicular bisector of AB. Label point C where the perpendicular bisector intersects AB. Mark a point D on the perpendicular bisector. Join AD and DB.

c) What kind of triangle have you drawn? How do you know? What does CD represent?

8.4

**5.** a) Draw obtuse $\angle PQR = 140°$. Use a ruler and compass to bisect $\angle PQR$.

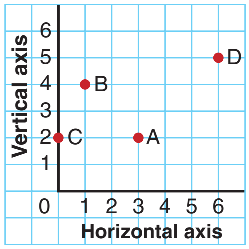
b) Draw acute $\angle CDE = 50°$. Use a different method to bisect $\angle CDE$.

c) How can you check that you have drawn each bisector correctly?

# 8.5 Graphing on a Coordinate Grid

**Focus** Identify and plot points in four quadrants of a coordinate grid.

You have plotted points with whole-number coordinates on a grid.
Point A has coordinates (3, 2).
What are the coordinates of point B? Point C? Point D?

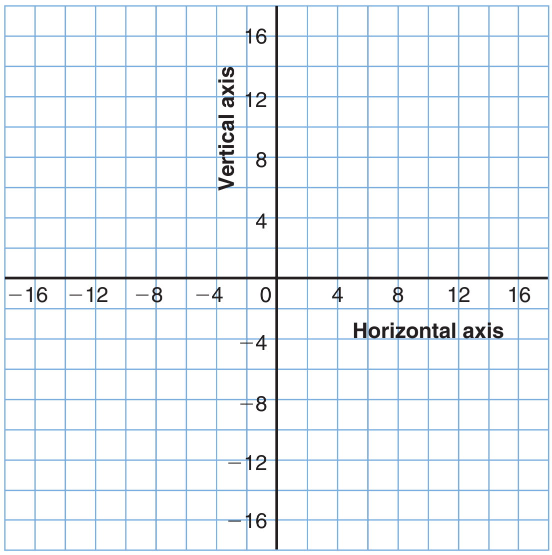



A vertical number line and a horizontal number line intersect at right angles at 0.
This produces a grid on which you can plot points with integer coordinates.

## Explore

You will need grid paper and a ruler. Copy this grid.

- Plot these points: A(14, 0), B(6, 2), C(8, 8), D(2, 6), E(0, 14)
  Join the points in order.
  Draw a line segment from each point to the origin.
- Reflect the shape in the vertical axis.
  Draw its image.
  Write the coordinates of each vertex of the image.
- Reflect the original shape and the image in the horizontal axis.
  Draw the new image.
  Write the coordinates of each vertex of the new image.




Your design should be symmetrical about the horizontal and vertical axes.
Describe the design. What shapes do you see?

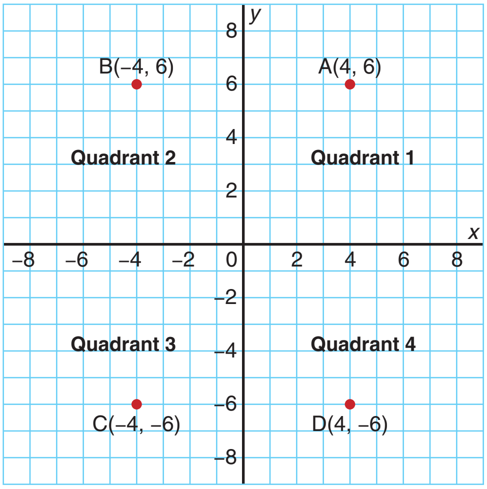
## Reflect & Share

Compare your design and its coordinates with those of another pair of classmates.
Describe any patterns you see in the coordinates of corresponding points.

## Connect

A vertical number line and a horizontal number line that intersect at right angles at 0 form a **coordinate grid**.
The horizontal axis is the ***x*-axis**.
The vertical axis is the ***y*-axis**.
The axes meet at the **origin**, (0, 0).
The axes divide the plane into four **quadrants**.
They are numbered counterclockwise.

This coordinate grid is also called a *Cartesian plane*.




We do not need arrows on the axes.

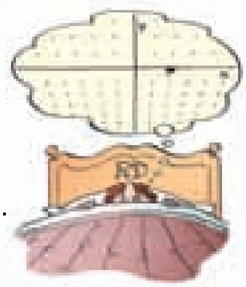
A pair of coordinates is called an *ordered pair*.

In Quadrant 1, to plot point A, start at 4 on the *x*-axis and move up 6 units.
Point A has coordinates (4, 6).
In Quadrant 2, to plot point B, start at −4 on the *x*-axis and move up 6 units.
Point B has coordinates (−4, 6).
In Quadrant 3, to plot point C, start at −4 on the *x*-axis and move down 6 units.
Point C has coordinates (−4, −6).
In Quadrant 4, to plot point D, start at 4 on the *x*-axis and move down 6 units.
Point D has coordinates (4, −6).

We do not have to include a + sign for a positive coordinate.

### Math Link

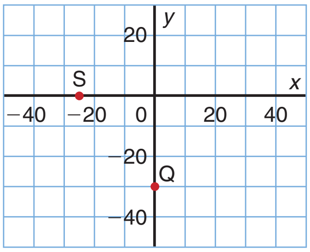
**History**
René Descartes lived in the 17th century.
He developed the coordinate grid.
It is named the Cartesian plane in his honour.
There is a story that René was lying in bed and watching a fly on the ceiling.
He invented coordinates as a way to describe the fly's position.



## Example

a) Write the coordinates of each point.

i) Q    ii) S




Notice that each grid square represents 10 units.

b) Plot each point on a grid.

i) F(0, −15)    ii) G(−40, 0)

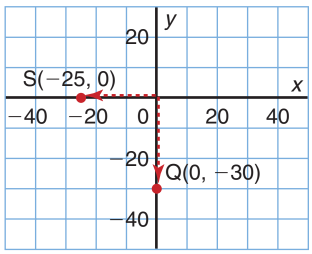
## A Solution

a) Start at the origin each time.

i) To get to Q, move 0 units right and 30 units down.
So, the coordinates of Q are (0, −30).

ii) To get to S, move 25 units left and 0 units down.
So, the coordinates of S are (−25, 0).

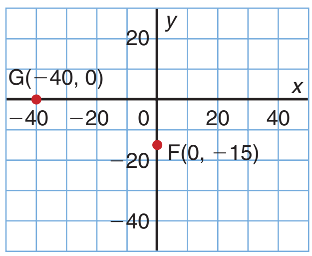
Remember, first move left or right, then up or down.




Point S is halfway between –20 and –30 on the *x*-axis.

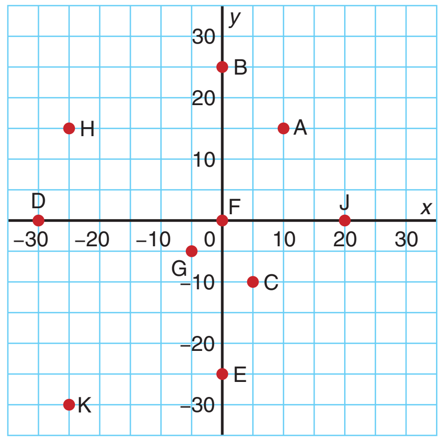
b) i) F(0, −15)
Since there is no movement left or right,
point F lies on the *y*-axis.
Start at the origin.
Move 15 units down the *y*-axis. Mark point F.
It is halfway between −10 and −20.

ii) G(−40, 0)
Start at −40 on the *x*-axis.
Since there is no movement up or down,
point G lies on the *x*-axis. Mark point G.



## Practice

1. What is the scale on each axis?
   Write the coordinates of each point from A to K.




2. Use the coordinate grid to the right.
   Which points have:
   a) *x*-coordinate 0?
   b) *y*-coordinate 0?
   c) the same *x*-coordinate?
   d) the same *y*-coordinate?
   e) equal *x*- and *y*-coordinates?
   f) *y*-coordinate 2?

3. Draw a coordinate grid. Look at the ordered pairs below.
   Label the axes. How did you choose the scale?
   Plot each point.

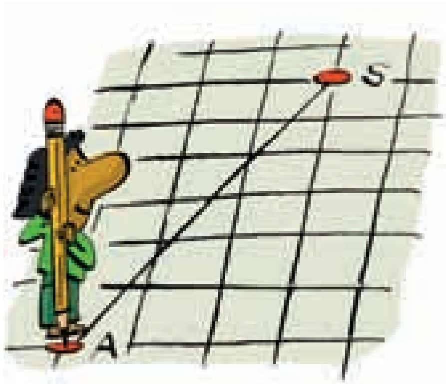
   a) A(30, −30) b) B(25, 0) c) C(−10, 35)
   d) D(−15, 40) e) E(15, 5) f) F(0, −20)
   g) O(0, 0) h) H(−20, −5) i) I(−40, 0)

   Which point is the origin?

4. How could you use the grid in question 3 to plot these points?
   a) K(3, 5) b) P(−10, 2) c) R(−7, −8)

5. Which quadrant has all negative coordinates? All positive coordinates?
   Both positive and negative coordinates?

6. a) Plot these points: A(0, 5), B(−1, 4), C(−1, 3), D(−2, 3), E(−3, 2), F(−2, 1), G(−1, 1), H(−1, 0), J(0, −1), K(1, 0), L(1, 1), M(2, 1), N(3, 2), P(2, 3), R(1, 3), S(1, 4)
   b) Join the points in order. Then join S to A.
   c) Describe the shape you have drawn.




7. Draw a design on a coordinate grid.
   Each vertex should be at a point where grid lines meet.
   List the points used to make the design, in order.
   Trade lists with a classmate.
   Use the list to draw your classmate's design.

**8.** Use a 1-cm grid.

a) Plot the points A(−3, 2) and B(5, 2).
Join the points to form line segment AB.
What is the horizontal distance between A and B?
How did you find this distance?

b) Plot the points C(3, −4) and D(3, 7).
Join the points to form line segment CD.
What is the vertical distance between C and D?
How did you find this distance?

**9.** Use question 8 as a guide.
Plot 2 points that lie on a horizontal or vertical line.
Trade points with a classmate.
Find the horizontal or vertical distance between your classmate's points.

**10.** **Assessment Focus** Use a coordinate grid.
How many different parallelograms can you draw that have area 12 square units?
For each parallelogram you draw, label its vertices.

**11.** a) Plot these points: K(−15, 20), L(5, 20), M(5, −10)

b) Find the coordinates of point N that forms rectangle KLMN.

**12.** a) Plot these points on a grid: A(16, −14), B(−6, 12), and C(−18, −14).
Join the points.
What scale did you use? Explain your choice.

b) Find the area of △ABC.

**13.** **Take It Further** The points A(−4, 4) and B(2, 4) are two vertices of a square.
Plot these points on a coordinate grid.
What are the coordinates of the other two vertices?
Find as many different answers as you can.

## Reflect

How did your knowledge of integers help you plot points on a Cartesian plane?

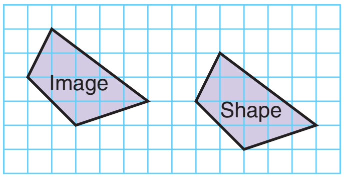
# 8.6 Graphing Translations and Reflections

**Focus** Graph translation and reflection images on a coordinate grid.

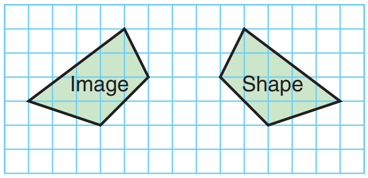
Recall that a translation slides a shape in a straight line. When the shape is on a square grid, the translation is described by movements right or left, and up or down.

**A translation and a reflection are transformations.**

Which translation moved this shape to its image?




A shape can also be reflected, or flipped, in a mirror line. Where is the mirror line that relates this shape and its image?




## Explore

You will need 0.5-cm grid paper and a ruler.
Draw axes on the grid paper to get 4 quadrants.
Use the whole page. Label the axes.
Draw and label a quadrilateral.
Each vertex should be where the grid lines meet.

- ➤ Translate the quadrilateral. Draw and label the translation image. What do you notice about the quadrilateral and its image?
- ➤ Choose an axis. Reflect the quadrilateral in this axis. Draw and label the reflection image. What do you notice about the quadrilateral and its image?
- ➤ Trade your work with that of a classmate. Identify your classmate's translation. In which axis did your classmate reflect?

### Reflect & Share

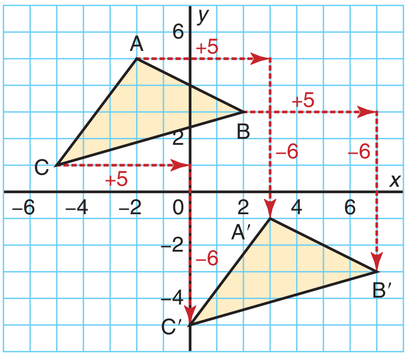
Did you correctly identify each transformation? Explain.
If not, work with your classmate to find the correct transformations.

## Connect

➤ To translate △ABC 5 units right and 6 units down:
Begin at vertex A(−2, 5).
Move 5 units right and 6 units down to point A′(3, −1).
From vertex B(2, 3), move 5 units right and 6 units down to point B′(7, −3).
From vertex C(−5, 1), move 5 units right and 6 units down to point C′(0, −5).

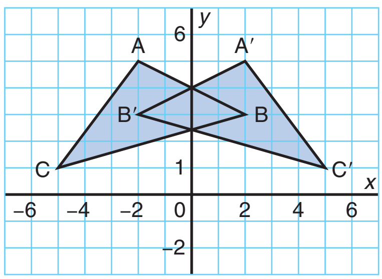
We read A′ as "A prime."

Each vertex of the image is labelled with a prime symbol.




Then, △A′B′C′ is the image of △ABC after a translation 5 units right and 6 units down.
△ABC and △A′B′C′ are congruent.

➤ To reflect △ABC in the *y*-axis:
Reflect each vertex in turn.
The reflection image of A(−2, 5) is A′(2, 5).
The reflection image of B(2, 3) is B′(−2, 3).
The reflection image of C(−5, 1) is C′(5, 1).




Then, △A′B′C′ is the image of △ABC after a reflection in the *y*-axis.
△ABC and △A′B′C′ are congruent.
The triangles have different orientations: we read △ABC clockwise; we read △A′B′C′ counterclockwise.

We could use a Mira to check the reflection.

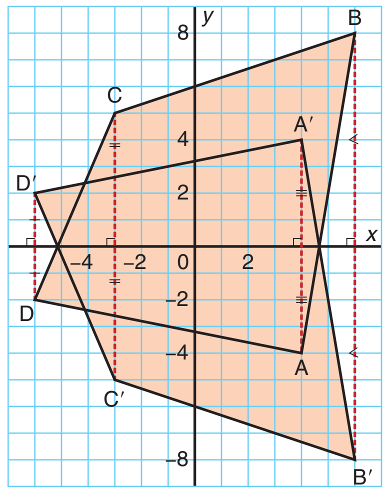
## Example

a) Plot these points: A(4, −4), B(6, 8), C(−3, 5), D(−6, −2)
Join the points to draw quadrilateral ABCD.
Reflect the quadrilateral in the *x*-axis.
Draw and label the reflection image A′B′C′D′.

b) What do you notice about the line segment joining each point to its reflection image?

## A Solution

a) To reflect quadrilateral ABCD in the *x*-axis:
Reflect each vertex in turn.
The reflection image of A(4, −4) is A′(4, 4).
The reflection image of B(6, 8) is B′(6, −8).
The reflection image of C(−3, 5) is C′(−3, −5).
The reflection image of D(−6, −2) is D′(−6, 2).



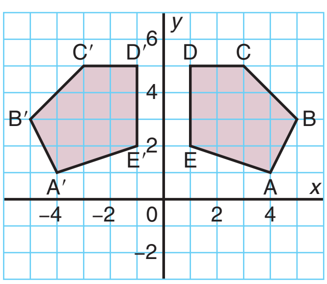

b) The line segments AA′, BB′, CC′, DD′ are vertical.
The *x*-axis is the perpendicular bisector of each line segment.
That is, the *x*-axis divides each line segment into 2 equal parts, and the *x*-axis intersects each line segment at right angles.

In *Practice* question 7, you will investigate a similar reflection in the *y*-axis.

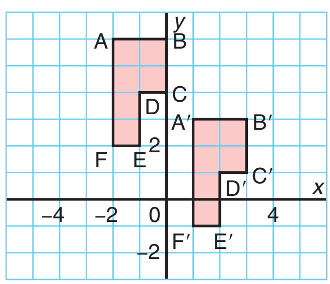
## Practice

**1.** Identify each transformation. Explain your reasoning.

a)




b)

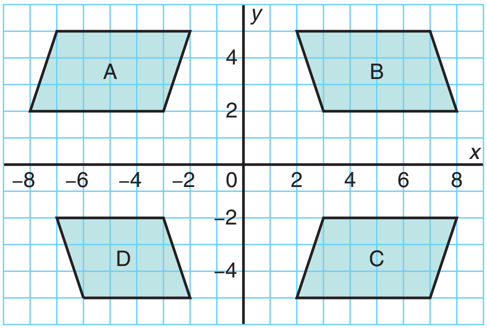


**2.** Describe the horizontal and vertical distance required to move each point to its image.

**a)** A(5, −3) to A′(2, 6) **b)** B(−3, 0) to B′(−5, −3) **c)** C(2, −1) to C′(4, 3)

**d)** D(−1, 2) to D′(−4, 0) **e)** E(3, 3) to E′(−3, 3) **f)** F(4, −2) to F′(4, 2)

**3.** The diagram shows 4 parallelograms.

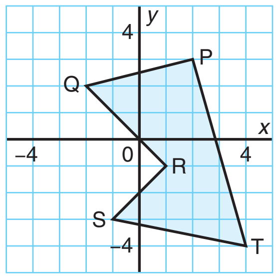



**a)** Are any 2 parallelograms related by a translation? If so, describe the translation.

**b)** Are any 2 parallelograms related by a reflection? If so, describe the reflection.

**4.** Copy this pentagon on grid paper.
Write the coordinates of each vertex.
After each transformation:

- Write the coordinates of the image of each vertex.
- Describe the positional change of the vertices of the pentagon.




**a)** Draw the image after a translation 3 units left and 2 units up.

**b)** Draw the image after a reflection in the *x*-axis.

**c)** Draw the image after a reflection in the *y*-axis.

**5.** Plot these points on a coordinate grid:
A(1, 3), B(3, −2), C(−2, 5), D(−1, −4), E(0, −3), F(−2, 0)

**a)** Reflect each point in the *x*-axis.
Write the coordinates of each point and its reflection image.
What patterns do you see in the coordinates?

**b)** Reflect each point in the *y*-axis.
Write the coordinates of each point and its reflection image.
What patterns do you see in the coordinates?

**c)** How could you use the patterns in parts a and b to check that you have drawn the reflection image of a shape correctly?

**6.** a) Plot the points in question 5.
Translate each point 4 units left and 2 units down.
b) Write the coordinates of each point and its translation image.
What patterns do you see in the coordinates?
c) How could you use these patterns to write the coordinates of an image point after a translation, without plotting the points?

**7.** a) Plot these points on a coordinate grid: P(1, 4), Q(−3, 4), R(−2, −3), S(5, −1)
Join the points to draw quadrilateral PQRS.
Reflect the quadrilateral in the *y*-axis.
b) What do you notice about the line segment joining each point to its image?

**8.** **Assessment Focus**

a) Plot these points on a coordinate grid:
A(2, 4), B(4, 4), C(4, 2), D(6, 2), E(6, 6)
Join the points to draw polygon ABCDE.
b) Translate the polygon 4 units right and 6 units up.
Write the coordinates of each vertex of the image polygon A′B′C′D′E′.
c) Reflect the image polygon A′B′C′D′E′ in the *y*-axis.
Write the coordinates of each vertex of the image polygon A″B″C″D″E″.
d) How does polygon A″B″C″D″E″ compare with polygon ABCDE?

**When there are 2 transformation images, we use a "double" prime notation for the vertices of the second image.**

**9.** a) Plot these points on a coordinate grid: F(−5, 8), G(0, 8), H(−1, 5), J(−5, 5)
Join the points to draw trapezoid FGHJ.
b) Translate the trapezoid 2 units right and 1 unit down.
c) Translate the image trapezoid F′G′H′J′ 2 units right and 1 unit down.
d) Repeat part c four more times for each resulting image trapezoid.
e) Describe the translation that moves trapezoid FGHJ to the final image.

**10.** **Take It Further** Draw a shape and its image that could represent a translation and a reflection. What attributes does the shape have?

## Reflect

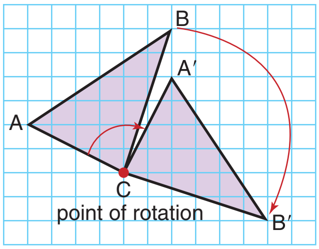
How is a translation different from a reflection?
How are these transformations alike?

# 8.7 Graphing Rotations

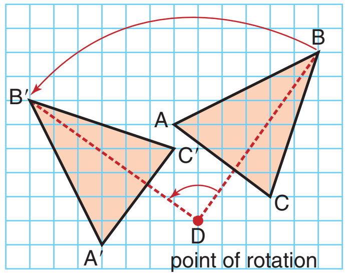
**Focus** Graph rotation images on a coordinate grid.

Recall that a rotation turns a shape about a point of rotation.
The rotation may be clockwise or counterclockwise.
The point of rotation may be:

On the shape




Off the shape




How would you describe each rotation?

## Explore

You will need 0.5-cm grid paper, tracing paper, a protractor, and a ruler.
Draw axes on the grid paper to get 4 quadrants.
Place the origin at the centre of the paper.
Label the axes.
Plot these points: O(0, 0), B(5, 3), and C(5, 4)
Join the points in order, then join C to O.
Use the origin as the point of rotation.

- Rotate the shape 90° counterclockwise. Draw its image.
- Rotate the original shape 180° counterclockwise. Draw its image.
- Rotate the original shape 270° counterclockwise. Draw its image.

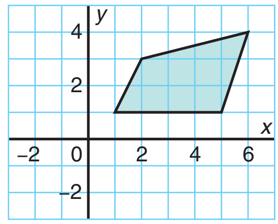
What do you notice about the shape and its 3 images?
What have you drawn?

## Reflect & Share

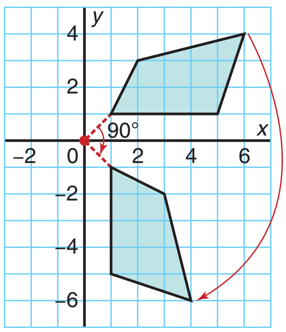
Compare your work with that of another pair of classmates.
What strategies did you use to measure the rotation angle?
Would the images have been different if you had rotated clockwise instead of counterclockwise? Explain your answer.

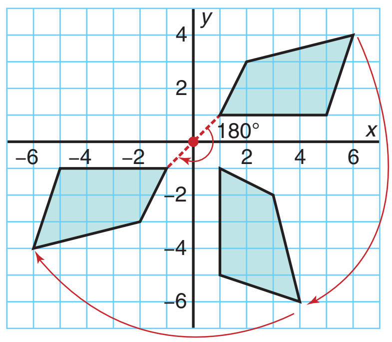
## Connect

To rotate the shape at the right clockwise:



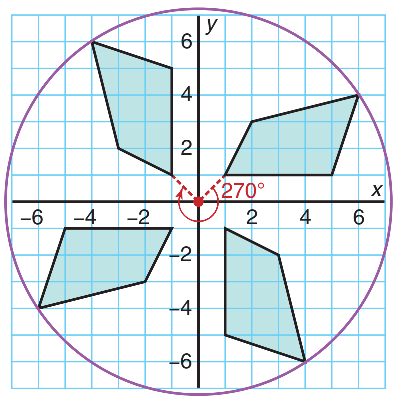

➤ Trace the shape and the axes.
Label the positive *y*-axis on the tracing paper.
Rotate the tracing paper clockwise about the origin until the positive *y*-axis coincides with the positive *x*-axis.
With a sharp pencil, mark the vertices of the image.
Join the vertices to draw the image after a 90° clockwise rotation about the origin, below left.




➤ Place the tracing paper so the shape coincides with its image.
Rotate the tracing paper clockwise about the origin until the positive *y*-axis coincides with the negative *y*-axis.
Mark the vertices of the image.
Join the vertices to draw the image of the original shape after a 180° clockwise rotation about the origin, above right.

➤ Place the tracing paper so the shape coincides with its second image.
Rotate the tracing paper clockwise about the origin until the positive *y*-axis coincides with the negative *x*-axis.
Mark, then join, the vertices of the image.
This is the image after a 270° clockwise rotation about the origin.




All 4 quadrilaterals are congruent.
A point and all its images lie on a circle, centre the origin.

## Example

a) Plot these points: B(−5, 6), C(−3, 4), D(−8, 2)
Join the points to draw △BCD.
Rotate △BCD 90° about the origin, O.
Draw and label the rotation image △B′C′D′.

b) Join C, D, C′, D′ to O.
What do you notice about these line segments?

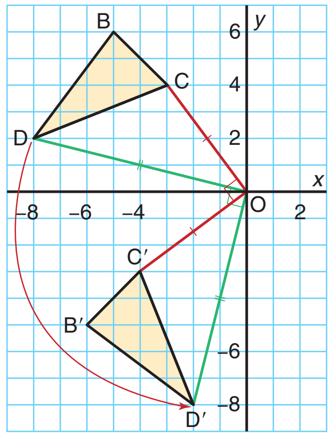
> A counterclockwise rotation is shown by a positive angle such as +90°, or 90°. A clockwise rotation is shown by a negative angle such as −90°.

## A Solution

A rotation of 90° is a counterclockwise rotation.

a) Use tracing paper to draw the image △B′C′D′.
Rotate the paper counterclockwise until the positive *y*-axis coincides with the negative *x*-axis.
After a rotation of 90° about the origin:
B(−5, 6) → B′(−6, −5)
C(−3, 4) → C′(−4, −3)
D(−8, 2) → D′(−2, −8)

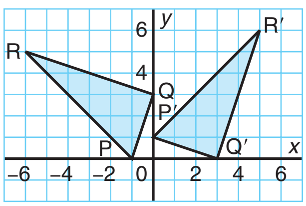
b) From the diagram, OC = OC′ and OD = OD′
∠COC′ = ∠DOD′ = 90°



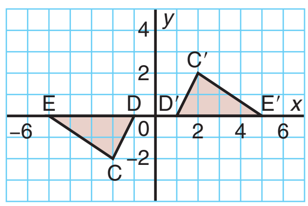

## Practice

**1.** Each grid shows a shape and its rotation image.
Identify the angle and direction of rotation, and the point of rotation.

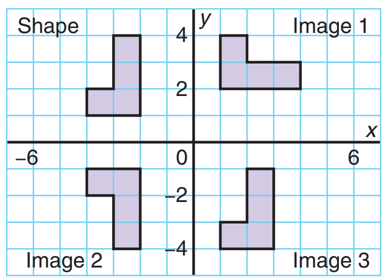
a)



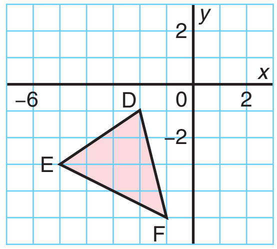

b)




**2.** Identify the transformation that moves the shape in Quadrant 2 to each image. Explain how you know.



3. a) Copy △DEF on grid paper.
Write the coordinates of each vertex.
After each rotation:
- Write the coordinates of the image of each vertex.
- Describe the positional change of the vertices of the triangle.




b) Rotate △DEF −90° about the origin to its image △D′E′F′.
c) Rotate △DEF +270° about the origin to its image △D″E″F″.
d) What do you notice about the images in parts b and c?
Do you think you would get a similar result with any shape that you rotate −90° and +270°? Explain.

4. Plot each point on a coordinate grid:
A(2, 5), B(−3, 4), C(4, −1)
a) Rotate each point 180° about the origin O to get image points A′, B′, C′.
Write the coordinates of each image point.
b) Draw and measure:
i) OA and OA′ ii) OB and OB′ iii) OC and OC′
What do you notice?
c) Measure each angle.
i) ∠AOA′ ii) ∠BOB′ iii) ∠COC′
What do you notice?
d) Describe another rotation of A, B, and C that would result in the image points A′, B′, C′.

5. Repeat question 4 for a rotation of −90° about the origin.

6. **Assessment Focus**
a) Plot these points on a coordinate grid:
A(6, 0), B(6, 2), C(5, 3), D(4, 2), E(2, 2), F(2, 0)
Join the points to draw polygon ABCDEF.
b) Translate the polygon 6 units left and 2 units up.
Write the coordinates of each vertex of the image polygon A′B′C′D′E′F′.
c) Rotate the image polygon A′B′C′D′E′F′ 90° counterclockwise about the origin.
Write the coordinates of each vertex of the image polygon A″B″C″D″E″F″.
d) How does polygon A″B″C″D″E″F″ compare with polygon ABCDEF?

**7.** Draw a large quadrilateral in the 3rd quadrant.

a) Rotate the quadrilateral 180° about the origin.

b) Reflect the quadrilateral in the *x*-axis.
Then reflect the image in the *y*-axis.

c) What do you notice about the image in part a and the second image in part b?
Do you think you would get a similar result if you started:
**i)** with a different shape? **ii)** in a different quadrant?
Investigate to find out. Write about what you discover.

**8.** a) Plot these points on a coordinate grid:
R(−1, −1), S(−1, 4), T(2, 4), U(2, −1)
Join the points to draw rectangle RSTU.

b) Choose a vertex to use as the point of rotation.
Rotate the rectangle 90° counterclockwise.

c) Repeat part b two more times for each image rectangle.

d) Describe the pattern you see in the rectangles.

e) Is there a transformation that moves rectangle RSTU to the final image directly? Explain.

**9.** **Take It Further** Plot these points: C(2, 6), D(3, −3), E(5, −7)

a) Reflect △CDE in the *x*-axis to its image △C′D′E′.
Rotate △C′D′E′ −90° about the origin to its image △C″D″E″.

b) Rotate △CDE −90° about the origin to its image △PQR.
Reflect △PQR in the *x*-axis to its image △P′Q′R′.

c) Do the final images in parts a and b coincide?
Explain your answer.

## Reflect

When you see a shape and its transformation image on a grid, how do you know what type of transformation it is?
Include examples in your explanation.

# Using a Computer to Transform Shapes

Geometry software can be used to transform shapes.
Use available geometry software.

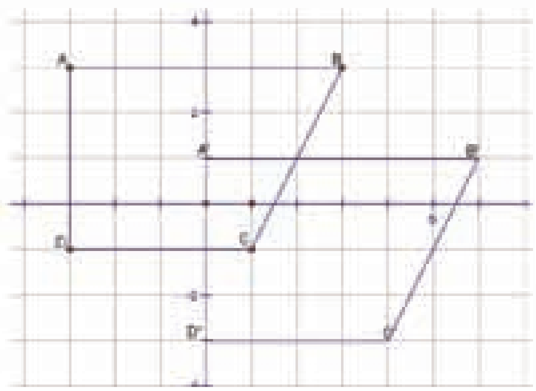
Open a new sketch. Check that the distance units are centimetres. Display a coordinate grid.

Should you need help at any time, use the software's Help Menu.

## Translating a Shape

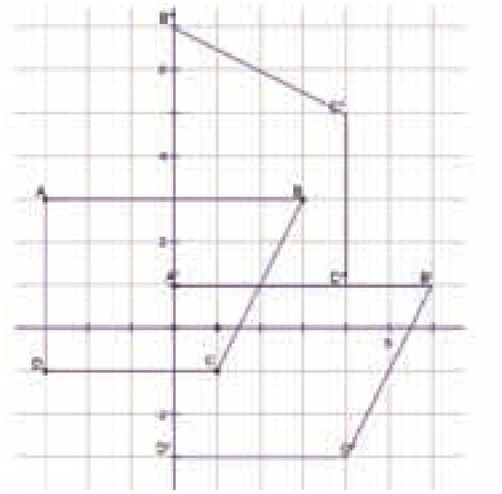
Construct a quadrilateral ABCD.
Record the coordinates of each vertex.

Select the quadrilateral. Use the software to translate the quadrilateral 3 units right and 2 units down. Record the coordinates of each vertex of the translation image A′B′C′D′.



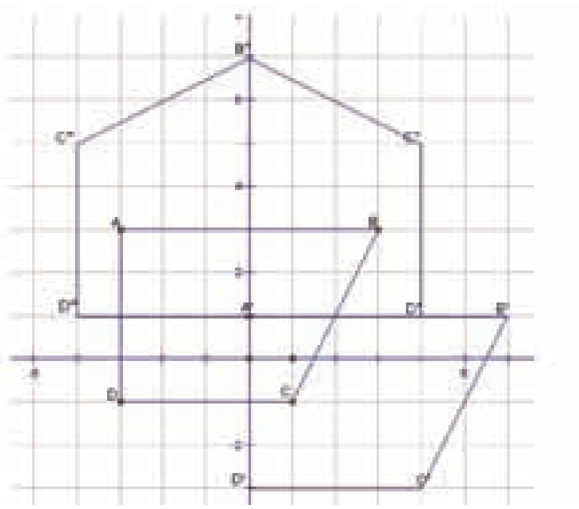
### Rotating a Shape

Use the image quadrilateral A′B′C′D′.
Select a vertex of the quadrilateral as the point of rotation. Select quadrilateral A′B′C′D′.
Rotate the quadrilateral 90° counterclockwise.
Record the coordinates of each vertex of the rotation image A″B″C″D″.



### Reflecting a Shape

Use the image quadrilateral A″B″C″D″.
Select one side of the quadrilateral as the mirror line.
Select quadrilateral A″B″C″D″.
Reflect the quadrilateral in the mirror line.
Record the coordinates of each vertex of the reflection image A‴B‴C‴D‴.



## ✓ Check

Create another shape.
Use any or all of the transformations above to make a design that covers the screen.
Colour your design to make it attractive. Print your design.

# Making a Study Card

There is a lot of important information in a math unit.

A study card can help you organize and review this information.

### How to Start

Read through each lesson in this unit.
As you read, make a study sheet by writing down:

- the main ideas of each lesson
- any new words and what they mean
- any formulas
- one or two examples per lesson
- any notes that might help you remember the concepts

### Making a Study Card

- Read over your study sheet.
- Decide which information to include on your study card.
  Only include information you need help remembering.
- Put the information on a recipe card.
  If you have too much information to fit on one card, use two or more cards.

Reading and Writing in Math

### Using Your Study Cards

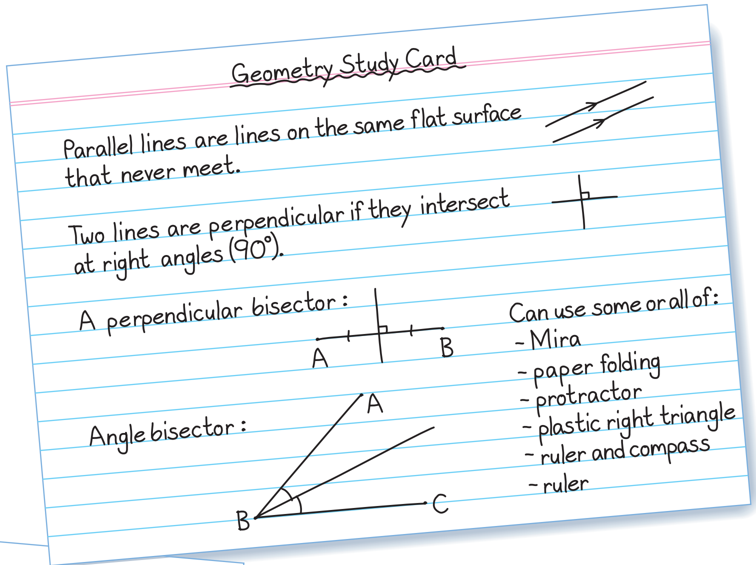
- Use your study cards as you work through the Unit Review.
- When you have finished, you should have some information on your study cards that you do not need help remembering anymore.
- Make a final study card using only one recipe card.

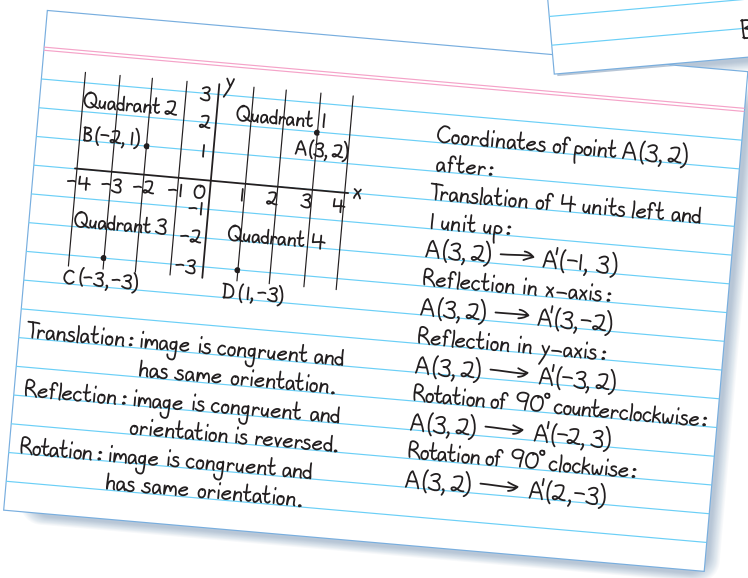
Compare your study card with that of a classmate.
What information does your classmate have that you do not have?
Should everyone's study card be the same? Explain.

The more you use your study card, the less you are going to need it.

Here is an example of a study card for Geometry.

Your study card is your own.
Make it so it helps you.
It may be different from your classmate's study card.




# Unit Review

## What Do I Need to Know?

### ✓ Parallel lines

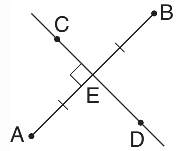
are lines on the same flat surface that never meet.

### ✓ Perpendicular lines

Two lines are *perpendicular* if they intersect at right angles.

### ✓ Perpendicular Bisector

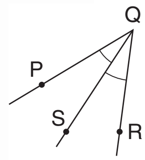
The *perpendicular bisector* of a line segment is drawn at right angles to the segment and divides the segment into two equal parts. Line segment CD is the perpendicular bisector of segment AB.




AE = EB
∠AEC = ∠CEB = 90°

### ✓ Bisector of an Angle

The *bisector of an angle* divides the angle into two equal angles. Line segment QS is the bisector of ∠PQR.




∠PQS = ∠SQR

### ✓ Transformations on a Coordinate Grid

A point or shape can be:

- translated (slid)
- reflected in the *x*-axis or the *y*-axis (flipped)
- rotated about the origin (turned)

**Math Link**

**Art**

Origami is the Japanese name for the art of paper folding. Use the library or the Internet to get instructions to make a model. Fold a sheet of paper to make your chosen model. Then, unfold the paper and look at the creases. Label as many pairs of parallel line segments as you can. Do the same with perpendicular line segments, perpendicular bisectors, and angle bisectors.

## What Should I Be Able to Do?

LESSON

8.1

**1.** **a)** Draw line segment FG of length 5 cm.

**b)** Mark a point H above FG. Draw a line segment parallel to FG that passes through point H.

**c)** Mark a point J below FG. Draw a line segment parallel to FG that passes through point J.

**d)** Explain how you can check that the line segments you drew in parts b and c are parallel.

8.2

**2.** **a)** Draw line segment CD of length 12 cm.

**b)** Mark a point E above CD. Draw a line segment perpendicular to CD that passes through point E. Label point F where the line segment intersects CD.

**c)** Join CE and ED. What does EF represent?

8.3

**3.** **a)** Draw line segment AB. Fold the paper to construct the perpendicular bisector.

**b)** Draw line segment CD. Use a Mira to construct the perpendicular bisector.

**c)** Draw line segment EF. Use a ruler and compass to construct the perpendicular bisector.

**d)** Which of the three methods is most accurate? Justify your answer.

8.4

**4.** **a)** Draw acute $\angle BAC = 70°$. Fold the paper to construct the angle bisector.

**b)** Draw right $\angle DEF$. Use a Mira to construct the angle bisector.

**c)** Draw obtuse $\angle GHJ = 100°$. Use a ruler and compass to construct the angle bisector.

**d)** Which method is most accurate? Justify your answer.

8.5

**5.** **a)** On a coordinate grid, plot each point. Join the points in order. Then join D to A. How did you choose the scale?

A(−20, −20) B(30, −20)
C(15, 30) D(−35, 30)

**b)** Name the quadrant in which each point is located.

**c)** Identify the shape. Find its area.

**6.** Do not plot the points. In which quadrant is each point located? How do you know?

**a)** A(6, −4) **b)** B(−4, −2)
**c)** C(−3, 2) **d)** D(6, 4)

**7.** **a)** Find the horizontal distance between each pair of points.

**i)** A(−5, 1) and B(7, 1)
**ii)** C(−2, −3) and D(9, −3)

**b)** Find the vertical distance between each pair of points.

**i)** E(4, −5) and F(4, 3)
**ii)** G(−3, −6) and H(−3, 0)

LESSON

**8.** Plot each point on a coordinate grid: A(−1, −1), C(3, 1)
A and C are opposite vertices of a rectangle. Find the coordinates of the other 2 vertices.

8.6 **9.** a) Plot these points on a coordinate grid.
P(3, 1), Q(7, 1), R(5, 3), S(3, 3)
Join the points to draw trapezoid PQRS.
How do you know it is a trapezoid? Explain.
b) Translate the trapezoid 4 units right. Write the coordinates of each vertex of the image trapezoid P′Q′R′S′.
c) Reflect trapezoid P′Q′R′S′ in the *x*-axis.
Write the coordinates of each vertex of the image trapezoid P″Q″R″S″.
d) How does trapezoid P″Q″R″S″ compare with trapezoid PQRS?

**10.** Repeat question 9. This time, apply the reflection before the translation. Is the final image the same? Explain.

8.7 **11.** a) Plot these points on a coordinate grid:
A (−2, 3) B(−4, 0)
C(−2, −3) D(2, −3)
Join the points to draw quadrilateral ABCD.
b) Draw the image of quadrilateral ABCD after each transformation:
i) a translation 7 units left and 8 units up
ii) a reflection in the *x*-axis
iii) a rotation of +90° about the origin
c) How are the images alike? Different?

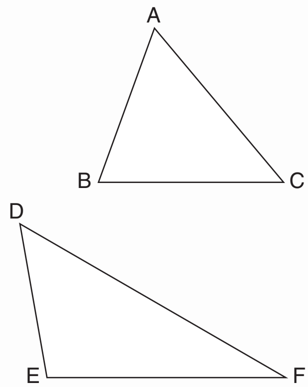
**12.** Use these points:
A(−2, 3), B(0, −1), and C(4, −3)
a) Suppose the order of the coordinates is reversed.
In which quadrant would each point be now?
Draw △ABC on a coordinate grid.
b) Which transformation changes the orientation of the triangle? Justify your answer with a diagram.
c) For which transformation is the image of AC perpendicular to AC? Justify your answer with a diagram.

**13.** a) Plot these points on a coordinate grid.
C(6, −3), D(−4, 3), E(6, 3)
Join the points to draw △CDE.
b) Translate △CDE 5 units left and 4 units up to image △C′D′E′.
c) Rotate △C′D′E′ +90° about the origin to image △C″D″E″.
d) How does △C″D″E″ compare with △CDE?

# Practice Test

**1.** Look around the classroom. Where do you see:
   **a)** parallel line segments?
   **b)** perpendicular line segments?
   **c)** perpendicular bisectors?
   **d)** angle bisectors?

**2.** Your teacher will give you a copy of each of these triangles.
   **a)** Construct the bisector of each angle in △ABC. Use a different method or tool for each bisector.
   **b)** Construct the perpendicular bisector of each side of △DEF. Use a different method or tool for each bisector.
   Describe each method or tool used.

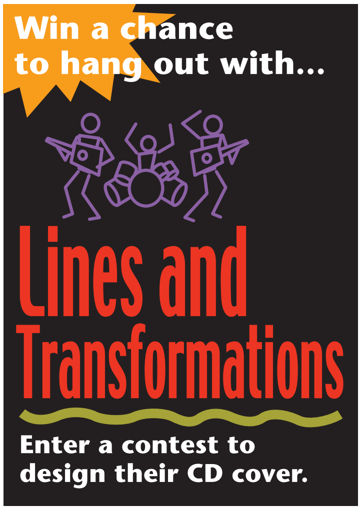



**3.** **a)** On a coordinate grid, draw a triangle with area 12 square units. Place each vertex in a different quadrant.
   **b)** Write the coordinates of each vertex.
   **c)** Explain how you know the area is 12 square units.
   **d)** Translate the triangle 6 units right and 3 units down. Write the coordinates of each vertex of the translation image.
   **e)** Reflect the triangle in the *y*-axis. Write the coordinates of each vertex of the reflection image.
   **f)** Rotate the triangle 90° clockwise about the origin. Write the coordinates of each vertex of the rotation image.

**4.** **a)** Plot these points on a coordinate grid.
   A(−2, 0), B(4, 0), C(3, 4), D(−1, 3)
   Join the points to draw quadrilateral ABCD.
   **b)** Translate quadrilateral ABCD 2 units left and 3 units down to the image quadrilateral A′B′C′D′.
   **c)** Translate quadrilateral A′B′C′D′ 6 units right and 7 units up to the image quadrilateral A″B″C″D″.
   **d)** Describe the translation that moves quadrilateral ABCD to quadrilateral A″B″C″D″.
   **e)** What would happen if the order of the translations was reversed? Explain your answer.

# Unit Problem

## Design the Cover




You have entered a contest to design the front and back covers of a CD for a new band called *Lines and Transformations*.

### Part 1

Your design for the front cover will be created on 4 pieces of paper.
It has to include:

- geometric shapes
- parallel line segments and perpendicular line segments
- geometric constructions

Work in a group of 4.
Brainstorm design ideas for the cover.
Sketch your cover. Show all construction lines.
Each person is responsible for one piece of the cover.
Make sure the pattern or design continues across a seam.
Draw your cover design.
Add colour to make it appealing.

Write about your design.
Explain your choice of design and how it relates to the geometric concepts of this unit.

## Check List

Your work should show:

- ✓ a detailed sketch of the front cover, including construction lines
- ✓ a design for the back cover, using transformations
- ✓ your understanding of geometric language and ideas
- ✓ accurate descriptions of construction methods and transformations used

## Part 2

Your design for the back cover will be created on a grid.
Each of you chooses a shape from your design for the front cover.
Draw the shape on a coordinate grid.
Use transformations to create a design with your shape.
Colour your design.

Write about your design.
Describe the transformations you used.
Record the coordinates of the original shape and 2 of the images.

## Reflect on Your Learning

Write 3 things you now know about parallel and perpendicular line segments that you did not know before.
What have you learned about transformations?

Investigation

# Integer Probability

**Materials:**

- four integer cards labelled −3, −2, +1, +3
- brown paper bag

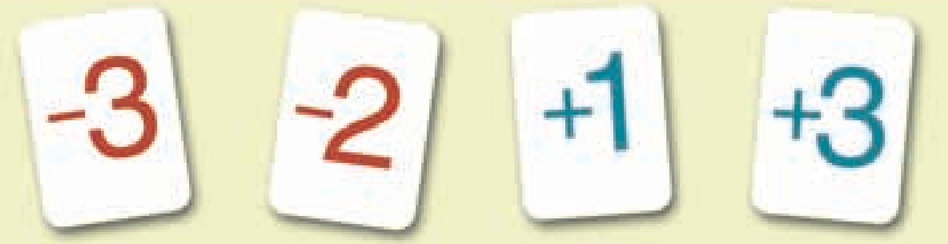
Work with a partner.

Four integer cards, labelled −3, −2, +1, and +3, are placed in a bag.

James draws three cards from the bag, one card at a time.
He adds the integers.

James predicts that because the sum of all four integers is negative, it is more likely that the sum of any three cards drawn from the bag will be negative.

In this *Investigation*, you will conduct James' experiment to find out if his prediction is correct.




## Part 1

➤ Place the integer cards in the bag.
Draw three cards and add the integers.
Is the sum negative or positive?
Record the results in a table.

| Integer 1 | Integer 2 | Integer 3 | Sum |
|---|---|---|---|
| | | | |

➤ Return the cards to the bag. Repeat the experiment until you have 20 sets of results.

- Look at the results in your table.
  Do the data support James' prediction?
  How can you tell?
- Combine your results with those of 4 other pairs of classmates.
  You now have 100 sets of results.
  Do the data support James' prediction?
  How can you tell?
- Use a diagram or other model to find the theoretical probability of getting a negative sum.
  Do the results match your experiment?
- Do you think the values of the integers make a difference?
  Find 4 integers (2 positive, 2 negative) for which James' prediction is correct.

## Part 2

Look at the results of your investigation in *Part 1*.

- If the first card James draws is negative, does it affect the probability of getting a negative sum?
  Use the results of *Part 1* to support your thinking.
- If the first card James draws is positive, does it affect the probability of getting a negative sum?
  Use the results of *Part 1* to support your thinking.