20.a) $\frac{1}{6}, 0.1\overline{6},$ about 16%

b) $\frac{100}{100}$, 1, 100%

c) 0, 0%

21.a) There are 48 possible outcomes: 1, 1; 1, 2; 1, 3; 1, 4; 1, 5; 1, 6; 2, 1; 2, 2; 2, 3; 2, 4; 2, 5; 2, 6; 3, 1; 3, 2; 3, 3; 3, 4; 3, 5; 3, 6; 4, 1; 4, 2; 4, 3; 4, 4; 4, 5; 4, 6; 5, 1; 5, 2; 5, 3; 5, 4; 5, 5; 5, 6; 6, 1; 6, 2; 6, 3; 6, 4; 6, 5; 6, 6; 7, 1; 7, 2; 7, 3; 7, 4; 7, 5; 7, 6; 8, 1; 8, 2; 8, 3; 8, 4; 8, 5; 8, 6

b) The outcome of rolling an octahedron does not depend on the outcome of rolling a die.

c) $\frac{4}{48} = \frac{1}{12}$, or $0.08\overline{3}$, or about 8.3%

24. Answers may vary. For example: If both coordinates are positive, the point is in Quadrant 1. If the *x*-coordinate is negative and the *y*-coordinate is positive, the point is in Quadrant 2. If both coordinates are negative, the point is in Quadrant 3. If the *x*-coordinate is positive and the *y*-coordinate is negative, the point is in Quadrant 4.
If the *x*-coordinate is 0, the point is on the *y*-axis. If the *y*-coordinate is 0, the point is on the *x*-axis.

25.a) Each grid square represents 5 units.

d) H

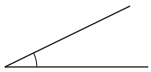
26.b) C′(–3, 9), D′ (1, 9), E′ (1, 3)

c) C″ (–3, –9), D″ (1, –9), E″ (1, –3)

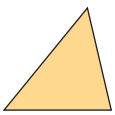
d) C‴(9, –3), D‴ (9, 1), E‴ (3, 1)

Illustrated Glossary

acute angle: an angle measuring less than 90°



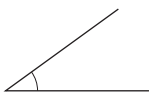
acute triangle: a triangle with three acute angles



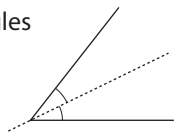
algebra tiles: a collective term for unit tiles and variable tiles

algebraic expression: a mathematical expression containing a variable: for example, $6x - 4$ is an algebraic expression

angle: formed by two rays from the same endpoint



angle bisector: the line that divides an angle into two equal angles



approximate: a number close to the exact value of an expression; the symbol $\doteq$ means "is approximately equal to"

area: the number of square units needed to cover a region

array: an arrangement in rows and columns

average: a single number that represents a set of numbers (see *mean, median,* and *mode*)

bar graph: a graph that displays data by using horizontal or vertical bars

bar notation: the use of a horizontal bar over a decimal digit to indicate that it repeats; for example, $1.\overline{3}$ means 1.333 333 …

base: the side of a polygon or the face of an object from which the height is measured

bisector: a line that divides a line segment or an angle into two equal parts

capacity: the amount a container can hold

Cartesian Plane: another name for a coordinate grid (see *coordinate grid*)

central angle: the angle between the two radii that form a sector of a circle

certain event: an event with probability 1, or 100%

chance: a description of a probability expressed as a percent

circle graph: a diagram that uses parts of a circle to display data

circumcentre: the point where the perpendicular bisectors of the sides of a triangle intersect (see *circumcircle*)

circumcircle: a circle drawn through all vertices of a triangle and with its centre at the circumcentre of the triangle

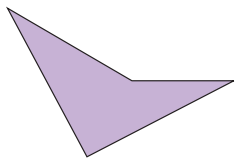
circumference: the distance around a circle, also known as the perimeter of the circle

common denominator: a number that is a multiple of each of the given denominators; for example, 12 is a common denominator for the fractions $\frac{1}{3}, \frac{5}{4}, \frac{7}{12}$

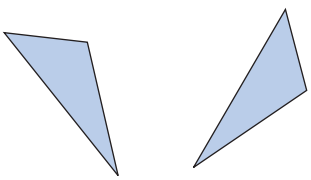
common factor: a number that is a factor of each of the given numbers; for example, 3 is a common factor of 15, 9, and 21

composite number: a number with three or more factors; for example, 8 is a composite number because its factors are 1, 2, 4, and 8

concave polygon: has at least one angle greater than 180°



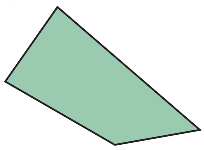
congruent: shapes that match exactly, but do not necessarily have the same orientation



consecutive numbers: integers that come one after the other without any integers missing; for example, 34, 35, 36 are consecutive numbers, so are $-2, -1, 0,$ and 1

constant term: the number in an expression or equation that does not change; for example, in the expression $4x + 3$, 3 is the constant term

convex polygon: has all angles less than 180°

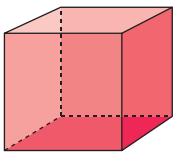


coordinate axes: the horizontal and vertical axes on a grid

coordinate grid: a two-dimensional surface on which a coordinate system has been set up

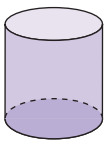
coordinates: the numbers in an ordered pair that locate a point on the grid (see *ordered pair*)

cube: an object with six congruent square faces



cubic units: units that measure volume

cylinder: an object with two parallel, congruent, circular bases

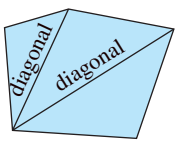


data: facts or information

database: an organized collection of facts or information, often stored on a computer

denominator: the term below the line in a fraction

diagonal: a line segment that joins two vertices of a shape, but is not a side



diameter: the distance across a circle, measured through its centre

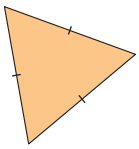
digit: any of the symbols used to write numerals; for example, in the base-ten system the digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

dimensions: measurements, such as length, width, and height

discount: the amount by which a price is reduced

equation: a mathematical statement that two expressions are equal

equilateral triangle: a triangle with three equal sides



equivalent: having the same value; for example, $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent fractions; 2:3 and 6:9 are equivalent ratios

estimate: a reasoned guess that is close to the actual value, without calculating it exactly

evaluate: to substitute a value for each variable in an expression

even number: a number that has 2 as a factor; for example, 2, 4, 6

event: any set of outcomes of an experiment

experimental probability: the probability of an event calculated from experimental results

expression: a mathematical phrase made up of numbers and/or variables connected by operations

factor: to factor means to write as a product; for example, $20 = 2 \times 2 \times 5$

formula: a rule that is expressed as an equation

fraction: an indicated quotient of two quantities

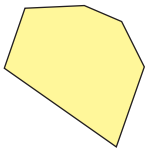
fraction strips: strips of paper used to model fractions

frequency: the number of times a particular number occurs in a set of data

greatest common factor (GCF): the greatest number that divides into each number in a set; for example, 5 is the greatest common factor of 10 and 15

height: the perpendicular distance from the base of a shape to the opposite side or vertex; the perpendicular distance from the base of an object to the opposite face or vertex

hexagon: a six-sided polygon



horizontal axis: the horizontal number line on a coordinate grid

image: the shape that results from a transformation

impossible event: an event that will never occur; an event with probability 0, or 0%

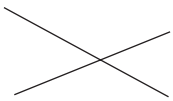
improper fraction: a fraction with the numerator greater than the denominator; for example, both $\frac{6}{5}$ and $\frac{5}{3}$ are improper fractions

independent events: two events in which the result of one event does not depend on the result of the other event

inspection: solving an equation by finding the value of the variable by using addition, subtraction, multiplication, and division facts

integers: the set of numbers
$\ldots -3, -2, -1, 0, +1, +2, +3, \ldots$

intersecting lines: lines that meet or cross; lines that have one point in common



inverse operation: an operation that reverses the result of another operation; for example, subtraction is the inverse of addition, and division is the inverse of multiplication

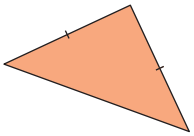
irrational number: a number that cannot be represented as a terminating or repeating decimal; for example, π

isosceles acute triangle: a triangle with two equal sides and all angles less than 90°

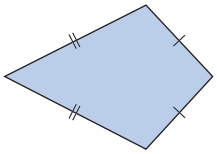
isosceles obtuse triangle: a triangle with two equal sides and one angle greater than 90°

isosceles right triangle: a triangle with two equal sides and a 90° angle

isosceles triangle: a triangle with two equal sides



kite: a quadrilateral with two pairs of equal adjacent sides



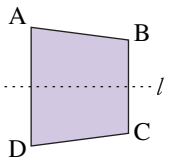
legend: part of a circle graph that shows what category each sector represents

linear relation: a relation whose points lie on a straight line

line graph: a graph that displays data by using points joined by line segments

line segment: the part of a line between two points on the line

line symmetry: a shape has line symmetry when it can be divided into 2 congruent parts, so that one part concides with the other part when the shape is folded at the line of symmetry; for example, line l is the line of symmetry for shape ABCD



lowest common multiple (LCM): the lowest multiple that is the same for two numbers; for example, the lowest common multiple of 12 and 21 is 84

magic square: an array of numbers in which the sum of the numbers in any row, column, or diagonal is always the same

magic sum: the sum of the numbers in a row, column, or diagonal of a magic square

mass: the amount of matter in an object

mean: the sum of a set of numbers divided by the number of numbers in the set

measure of central tendency: a single number that represents a set of numbers (see *mean, median,* and *mode*)

median: the middle number when data are arranged in numerical order; if there is an even number of data, the median is the mean of the two middle numbers

midpoint: the point that divides a line segment into two equal parts

mixed number: a number consisting of a whole number and a fraction; for example, $1\frac{1}{18}$ is a mixed number

mode: the number that occurs most often in a set of numbers

multiple: the product of a given number and a natural number; for example, some multiples of 8 are 8, 16, 24, …

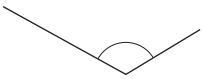
natural numbers: the set of numbers 1, 2, 3, 4, 5, …

negative number: a number less than 0

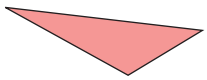
numerator: the term above the line in a fraction

numerical coefficient: the number by which a variable is multiplied; for example, in the expression $4x + 3$, 4 is the numerical coefficient

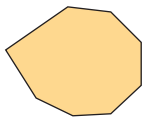
obtuse angle: an angle greater than 90° and less than 180°



obtuse triangle: a triangle with one angle greater than 90°



octagon: an eight-sided polygon



odd number: a number that does not have 2 as a factor; for example, 1, 3, 7

operation: a mathematical process or action such as addition, subtraction, multiplication, or division

opposite integers: two integers with a sum of 0; for example, $+3$ and -3 are opposite integers

ordered pair: two numbers in order, for example, (2, 4); on a coordinate grid, the first number is the horizontal coordinate of a point, and the second number is the vertical coordinate of the point

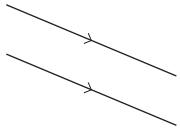
order of operations: the rules that are followed when simplifying or evaluating an expression

origin: the point where the *x*-axis and the *y*-axis intersect

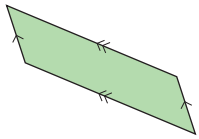
outcome: a possible result of an experiment or a possible answer to a survey question

outlier: a number in a set that is significantly different from the other numbers

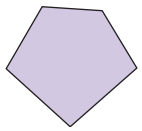
parallel lines: lines on the same flat surface that do not intersect



parallelogram: a quadrilateral with both pairs of opposite sides parallel



pentagon: a five-sided polygon



percent: the number of parts per 100; the numerator of a fraction with denominator 100

percent circle: a circle divided into 10 congruent sectors, with each sector further divided into 10 parts; each part is 1% of the circle

perimeter: the distance around a closed shape

perpendicular bisector: the line that is perpendicular to a line segment and divides the line segment into two equal parts

perpendicular lines: intersect at 90°

polygon: a closed shape that consists of line segments; for example, triangles and quadrilaterals are polygons

polyhedron (*plural,* polyhedra): an object with faces that are polygons

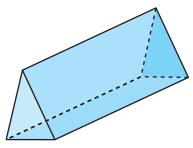
population: the set of all things or people being considered

positive number: a number greater than 0

prediction: a statement of what you think will happen

prime number: a whole number with exactly two factors, itself and 1; for example, 2, 3, 5, 7, 11, 29, 31, and 43

prism: an object that has two congruent and parallel faces (the *bases*), and other faces that are parallelograms

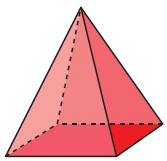


probability: the likelihood of a particular outcome; the number of times a particular outcome occurs, written as a fraction of the total number of outcomes

product: the result when two or more numbers are multiplied

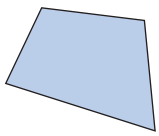
proper fraction: a fraction with the numerator less than the denominator; for example, $\frac{5}{6}$

pyramid: an object that has one face that is a polygon (the *base*), and other faces that are triangles with a common vertex



quadrant: one of four regions into which coordinate axes divide a plane

quadrilateral: a four-sided polygon



quotient: the result when one number is divided by another

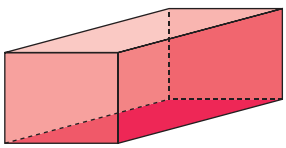
radius (*plural,* radii): the distance from the centre of a circle to any point on the circle

range: the difference between the greatest and least numbers in a set of data

ratio: a comparison of two or more quantities with the same unit

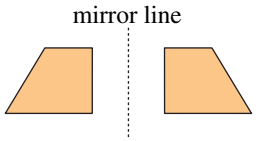
rectangle: a quadrilateral that has four right angles

rectangular prism: a prism that has rectangular faces



rectangular pyramid: a pyramid with a rectangular base

reflection: a transformation that is illustrated by a shape and its image in a mirror line



reflex angle: an angle between 180° and 360°

regular hexagon: a polygon that has six equal sides and six equal angles

regular octagon: a polygon that has eight equal sides and eight equal angles

regular polygon: a polygon that has all sides equal and all angles equal

related denominators: two fractions where the denominator of one fraction is a factor of the other; their lowest common denominator is the greater of the two denominators

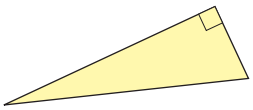
relation: a variable compared to an expression that contains the variable

repeating decimal: a decimal with a repeating pattern in the digits that follow the decimal point; it is written with a bar above the repeating digits; for example, $\frac{1}{11} = 0.\overline{09}$

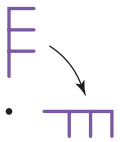
rhombus: a parallelogram with four equal sides

right angle: a 90° angle

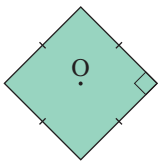
right triangle: a triangle that has one right angle



rotation: a transformation in which a shape is turned about a fixed point



rotational symmetry: a shape that coincides with itself in less than one full turn about its centre is said to have rotational symmetry; for example, a square has rotational symmetry



sample/sampling: a representative portion of a population

sample space: a list of all possible outcomes for an experiment that has independent events

scale: the numbers on the axes of a graph

scalene triangle: a triangle with all sides different

sector: part of a circle between two radii and the included arc

sector angle: see *central angle*

simplest form: a ratio with terms that have no common factors, other than 1; a fraction with numerator and denominator that have no common factors, other than 1

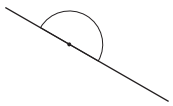
spreadsheet: a computer-generated arrangement of data in rows and columns, where a change in one value results in appropriate calculated changes in the other values

square: a rectangle with four equal sides

square number: the product of a number multiplied by itself; for example, 25 is the square of 5

statistics: the branch of mathematics that deals with the collection, organization, and interpretation of data

straight angle: an angle measuring 180°



surface area: the total area of the surface of an object

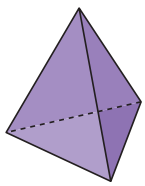
symmetrical: possessing symmetry (see *line symmetry* and *rotational symmetry*)

systematic trial: solving an equation by choosing a value for the variable, then checking by substituting

term: (of a fraction) the numerator or the denominator of the fraction

terminating decimal: a decimal with a certain number of digits after the decimal point; for example, $\frac{1}{8} = 0.125$

tetrahedron: an object with four triangular faces; a triangular pyramid

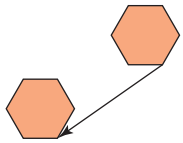


theoretical probability: the number of favourable outcomes written as a fraction of the total number of possible outcomes

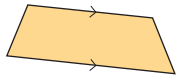
three-dimensional: having length, width, and depth or height

transformation: a translation, rotation, or reflection

translation: a transformation that moves a point or a shape in a straight line to another position on the same flat surface



trapezoid: a quadrilateral that has at least one pair of parallel sides



tree diagram: a diagram that resembles the roots or branches of a tree, used to count outcomes

triangle: a three-sided polygon

two-dimensional: having length and width, but no thickness, height, or depth

unit fraction: a fraction that has a numerator of 1

unit price: the price of one item, or the price of a particular mass or volume of an item

unit tile: a tile that represents $+1$ or -1

unrelated denominators: two fractions where the denominators have no common factors; their lowest common denominator is the product of the two denominators

variable: a letter or symbol representing a quantity that can vary

variable tile: a tile that represents a variable

vertex (*plural,* vertices): the corner of a shape or object

vertical axis: the vertical number line on a coordinate grid

volume: the amount of space occupied by an object

whole numbers: the set of numbers 0, 1, 2, 3, …

***x*-axis:** the horizontal number line on a coordinate grid

***y*-axis:** the vertical number line on a coordinate grid

zero pair: two opposite numbers whose sum is equal to zero

Index

A

adding integers, 78
addition equation, 56, 72
algebra, 22 *Math Link*
  solving equations with, 237, 238, 241–243, 248
algebra tiles, 38–41, 43
  solving equations with, 231–233, 248
algebraic expressions, 16, 17, 35, 36, 38–41, 43, 220
angle bisectors, 310–312, 334
area,
  of a circle, 148–150, 167
  of a parallelogram, 139–141, 144
  of a rectangle, 140, 150
  of a triangle, 143–145, 167
area models, 181
averages, 271–273
  investigating with spreadsheets, 276

B

balance-scale models, 226–228, 238, 242, 248
base, 144, 145, 149, 150
  of a parallelogram, 140, 167
Base Ten Blocks,
  dividing with, 104–106
  multiplying with, 100, 101
benchmarks for comparisons, 91
bisect, 306, 310

C

calculation errors, 155
Carroll diagram, 12
Cartesian plane, 316 *Math Link*
central angle, 161
certain event, 281, 292
chance, 280
circle,
  area of, 148–150, 167
  circumference, 133–135, 167
  diameter, 131
  investigating, 130, 131
  perimeter, 134
  radius (*pl.* radii), 130
circle graphs, 156–158, 161, 162, 167
  creating with spreadsheets, 165, 166
circumcentre, 309
circumcircle, 309
circumference, 133–135, 167
common denominator, 186, 187, 196, 197, 212
congruent, 321, 326
congruent sectors, 149
congruent shapes, 131
congruent triangles, 144
constant term, 17, 21
coordinate grid, 315–317, 334
coordinates, 315–317
copying errors, 154
Cuisenaire rods,
  modelling mixed numbers, 204, 205

D

decimals, 120
  adding and subtracting, 96–98
  comparing and ordering, 91–93
  dividing, 104–106
  from fractions, 86–88
  multiplying, 100, 101
  order of operations with, 108
  relating to fractions and percents, 111, 112
denominators,
  related and unrelated, 184
Descartes, René, 316 *Math Link*
diameter, 131, 167
digital roots, 174 *Investigation*
dividend, 105
divisibility, 6–8, 10–12
divisibility rules, 6–8, 10–12, 43
division,
  patterns in, 6–8, 10–12
division sentence, 104
divisor, 105
double prime symbol (″), 324

E

equations, 38–41, 220–223, 240–243, 248
  preserving equality of, 229, 248
  reading and writing, 35, 36, 43
  solving with algebra, 237, 238, 241–243, 248
  solving with integers, 231–234
  solving with models, 226–228
equivalent fractions, 182, 186, 187, 192, 212
  ordering fractions with, 92
errors, 154, 155
evaluate, 17, 43
experimental probability, 284, 286

F

factors, 8, 11
fraction circles, 179, 199, 200
fraction strips, 181–183
  adding fractions with, 188
  modelling mixed numbers, 201
  subtracting fractions with, 192, 195, 196
fractions, 120
  adding and subtracting, 212
  adding with models, 178, 179, 181–183
  adding with symbols, 186–188
  comparing and ordering, 91–93
  converting to decimals, 86–88

expressing probability, 280, 281
from circle graphs, 157, 158
relating to decimals and percents, 111, 112
subtracting with models, 191, 192
subtracting with symbols, 195–197
Frayer Model, 290
front-end estimation, 97, 98, 101

G

Games
All the Sticks, 289
Equation Baseball, 245
Packing Circles, 153
graphs,
circle, 156–158
coordinate grids, 315–317
showing relations with, 30–32

H

height, 144, 145, 150
of a parallelogram, 140, 167
hexagon, 178, 192
homework log, 76, 77

I

impossible event, 281, 292
improper fraction, 188, 197, 200, 201, 206, 212
independent events, 285, 292
input/output machine, 25, 26
inspection, 221–223, 234, 248
instructions, 118, 119
integers,
adding, 78
adding on a number line, 60–62, 78
adding with tiles, 56, 57, 78
negative, 52, 53, 56, 57, 60–62, 66–68, 72, 73, 78
opposite, 60, 72, 73, 78
positive, 52, 53, 56, 57, 60–62, 66–68, 72, 73, 78
representing, 52, 53
solving equations with, 231–234
subtracting, 78
subtracting on a number line, 71–73, 78
subtracting with tiles, 66–68, 78
intersecting lines, 303
irrational number, 134, 167

K

key words, 247

L

legend of a graph, 157
line segments, 301, 303–306, 311, 334
linear relations, 31, 32, 43

M

Math Link
Agriculture: Crop Circles, 152
Art, 334
History, 22, 316
Music, 185
Science, 225
Sports, 54
Your World, 120, 288
mean, 258–260, 264, 268, 271–273, 292
median, 263, 264, 268, 271–273, 292
Mira, 304, 305, 312
mixed numbers, 91, 188, 212
adding with, 199–201
subtracting with, 204–206
modelling with Cuisenaire rods, 204, 205
mode, 259, 260, 264, 268, 271–273, 292
models,
adding fractions with, 178, 179, 181–183
subtracting fractions with, 191, 192
multiples, 7, 8
multiplication fact, 174
Investigation

N

negative integer, 52, 56, 57, 60–62, 66–68, 72, 73, 78
notation errors, 155
number lines,
adding fractions on, 182, 183, 188
adding integers on, 60–62, 78
modelling mixed numbers, 201
ordering fractions on, 92, 93
relating decimals, fractions, and percents, 111, 112
subtracting fractions on, 192, 195, 196
subtracting integers on, 71–73, 78
numerical coefficient, 17

O

obtuse triangle, 144
opposite integers, 60, 72, 73, 78
order of operations, 26, 120
with decimals, 108
ordered pair (*see also* coordinates), 316
origin, 316
outcome, 284–286
outliers, 267, 268, 292

P

parallel lines, 300, 301, 334
parallelogram,
area of, 139–141, 144
Pattern Blocks,
subtracting fractions with, 191, 192
pattern rule, 20
patterns,
in decimals and fractions, 87
in division, 6–8, 10–12
in tables, 25–27
relationships in, 20–22
percent circles, 161, 162
percents, 120
expressing probability, 280, 281
from circle graphs, 156–158

relating to fractions and decimals, 111, 112
solving problems in, 114, 115
perimeter,
of a circle, 134
perpendicular bisectors, 306–308, 322, 334
perpendicular lines, 303, 304, 334
pi (π), 134, 135, 167
pie charts, 165
place-value charts,
ordering decimals on, 93
positive integers, 52, 56, 57, 60–62, 66–68, 72, 73, 78
pressure, 225 *Math Link*
prime number, 90
prime symbol (′), 321
probability, 279–281
experimental, 284, 286
theoretical, 282, 286
protractor, 301, 304

Q

quadrants, 316
quotient, 105

R

radius (*pl.* radii), 130, 167
range, 263, 264, 292
ratio,
expressing probability, 280, 281
rectangle,
area of, 140, 150
reflection, 320–322, 334
computing, 331
reflex angle, 312
related denominators, 184
relations, 21, 25–27, 43
graphing, 30–32
linear, 31, 32, 43
relationships,
in patterns, 20–22
in tables, 25–27
repeating decimals, 87, 88
rhombus, 178, 192, 306, 307, 311
rotations, 325–327, 334
computing, 331

S

sample space, 286
sector angles, 161
sector,
of a circle graph, 156
signatures, 2
simplest form, 182, 187
solutions,
verifying, 227, 228, 233, 238, 241
writing, 210, 211
spreadsheet software, 165, 166
investigating averages with, 276
study cards, 332, 333
subtracting integers, 78
subtraction, 71
subtraction equations, 67, 72
symbols,
adding fractions with, 186–188
subtracting fractions with, 195–197
systematic trial, 221–223, 248

T

tables,
patterns and relationships in, 25–27
terminating decimals, 87, 88
theoretical probability, 284, 286
thinking log, 14
tiles,
adding integers with, 56, 57, 78
subtracting integers with, 66–68, 78
time zones, 82 *Unit Problem*
transformations, 320, 334
translations, 320–322, 334
computing, 330
trapezoid, 178, 192
tree diagrams, 284–286
triangle, 178, 192
area of, 143–145, 167

U

unit fraction, 189
unit tile, 38
unlike denominators, 212
unrelated denominators, 184

V

variable tile, 38
variables, 16, 17, 21, 36, 43, 233
isolating, 39
Venn diagram, 8, 10
verifying solutions, 227, 228, 233, 238, 241

W

word problems, 246, 247
World of Work
Advertising Sales Representative, 209
Sports Trainer, 117
writing solutions, 210, 211

X

x-axis, 316, 322, 326, 327, 334

Y

y-axis, 316, 326, 334

Z

zero pairs, 52, 53, 56, 57, 66–68, 231–233